Review for the 3rd Exam.

Date of the exam: April 21, 2015 Tuesday
9:10 - 10:10 AM

Material: Chapters 6 & 7

Chapter 6

Know the following equations related to the Bohr Hydrogen atom:

1. \[ V_n \rightarrow \alpha \frac{e^2}{n} \quad \alpha = \frac{e^2}{4\pi \varepsilon_0 c} = \frac{1}{137.036} \]

2. \[ r_n \rightarrow n^2 \frac{\hbar}{mc} = n^2 a_0 \quad a_0 = \text{Bohr radius} = 0.0529 \text{ nm} \]

3. \[ E_n \rightarrow -\frac{1}{2} mc^2 \frac{\alpha^2}{n^2} = -\frac{E_R}{n^2} = -13.6 \text{ eV} \]

Know how to modify these equations for hydrogen-like atoms:

\[ \alpha \rightarrow Z \alpha \quad 1. \quad V_n \rightarrow \frac{Z \alpha e^2}{n} \quad 2. \quad r_n \rightarrow \frac{n^2 a_0}{Z} \quad 3. \quad E_n = -\frac{Z^2 E_R}{n^2} \]

Know how to modify these equation for the reduced mass

\[ \mu = \text{reduced mass} \quad \mu \text{ for (} e^-\text{, proton)} = \frac{m_e m_p}{m_e + m_p} \]

\[ \mu = \frac{m_e}{m_e/m_p + 1} = \frac{m_e}{\frac{1}{1836} + 1} = \frac{(1836)}{1837} m_e \approx m_e \]

\[ \mu \text{ for (} \mu^-\text{, proton)} = \frac{m\mu m_p}{m\mu + m_p} = \frac{207 m_e m_p}{207 m_e + m_p} \]

\[ \mu \text{ for (} \mu^-\text{, proton)} = \frac{207 m_e}{\frac{207 m_e}{m_p} + 1} = \frac{207 m_e}{207 + 1} \]

\[ \mu \text{ for (} \mu^-\text{, proton)} = \frac{207 m_e}{2043/1836} = 184 m_e \]
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Why do we calculate the reduced masses in terms of \( m_e \)?

Because the calculations for equations ① \& ③ are simplified.

For example: If we have a \( \mu^- \) orbiting a proton

\[
② \quad r_n = \frac{n^2 \hbar}{\mu c a_0} = \frac{n^2 \hbar}{186 \text{ MeV} c \alpha} = n^2 \alpha_0 = 0.0529 \text{ nm}
\]

\[
③ \quad E_n = -\frac{1}{2} \mu c^2 \frac{\alpha^2}{n^2} = -\frac{1}{2} \frac{(186 \text{ MeV}) c^2 \alpha^2}{n^2}
\]

\[
E_n = (-186) \left( \frac{1}{2} \frac{\text{MeV} c^2 \alpha^2}{n^2} \right) = -186 \left( \text{E}_R \right)
\]

Know how to calculate the wavelengths absorbed and emitted in the Bohr model.

Example: Find the wavelength emitted for \( n = 4 \rightarrow n = 2 \).

Cons. of energy \( \Rightarrow \quad E_4 = E_2 + \frac{hc}{\lambda} \quad \frac{hc}{\lambda} = E_4 - E_2 \)

\[
\frac{hc}{\lambda} = \frac{1}{2} \frac{\text{MeV} c^2 \alpha^2}{n^2} = \frac{1}{2} \frac{\text{MeV} c^2 \alpha^2}{\left( \frac{1}{4^2} - \frac{1}{2^2} \right)} = \frac{1}{2} \frac{\text{MeV} c^2 \alpha^2}{\left( \frac{1}{16} - \frac{1}{4} \right)} = \text{E}_R \left( \frac{3}{16} \right)
\]

\[
\lambda = \frac{hc}{\left( \frac{3}{16} \text{E}_R \right)} = 1240 \text{ eV} \cdot \text{nm} = 486.3 \text{ nm}
\]

⇒ Know how to calculate this in a hydrogen-like atom.

⇒ Know how to calculate this if reduced masses are present.

Rutherford Scattering:

You will be given the following equations: ① \& ②

\[
① \quad b = \text{impact parameter} = z z' \frac{a_0 \alpha \text{hc} \cot(\theta/2)}{2x}
\]
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\[ f_{\theta} = n t \pi b^2 \]  fraction scattered at angles > \theta

You should know how to calculate \( n t \):

\[
n = \frac{\text{# of nuclei}}{m^3} = \rho \left( \frac{\text{kg/m}^3}{\text{kg/m}^3} \right) N_A \left( \frac{\text{# of nuclei}}{\text{mol}} \right) \frac{M}{\text{(molecular mass kg/mol)}}
\]

\( t = \text{thickness (meters)} \)

\[ \text{Example: Thin target of gold. } \quad t = 8.00 \times 10^{-6} \text{ m}. \]

\[
\rho_{Au} = 19,300 \text{ kg/m}^3 \quad N_A = 6.022 \times 10^{23} \text{ nuclei/mol}
\]

\[
M = 197 \text{ g/mol} \rightarrow 197 \times 10^{-3} \text{ kg/mol}
\]

\[
nt = \left( \frac{\rho N_A}{M} \right) t = \frac{19,300 \left( 6.022 \times 10^{23} \right)}{197 \times 10^{-3}} \left( 8.00 \times 10^{-6} \right)
\]

\[
nt = 4.72 \times 10^{18} \text{ nuclei/m}^2
\]

\[ \text{Example: What fraction of 6 MeV } \alpha \text{-particles will scatter at angles > 10°?} \]

\[
f_{\theta} = n t \pi b^2 \quad b = 2Z \alpha \times \text{cot} \left( \frac{\theta}{2} \right) \frac{1}{2k}
\]

\[
b = 2(19) \frac{1}{137} \left( 197 \text{ MeV \cdot fm} \right) \text{cot} (5°) = 216 \text{ fm}
\]

\[
f_{10} = 4.72 \times 10^{23} \text{ nuclei/m}^2 \pi \left( 216 \times 10^{-15} \text{ m} \right)^2 = 0.0694 = 6.94\%
\]

\[ \text{Example: What is } f_{10} \text{ if the kinetic energy is doubled?} \]

If \( K \rightarrow 2K \) then \( b \rightarrow \frac{1}{2} b \) and \( f_{10} \rightarrow \frac{1}{4} f_{10} \)

\[
f_{10} = \frac{1}{4} (6.94\%) = 1.74\%.
\]
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Rutherford Atom continued:

You should know how to calculate the distance of closest approach for a "head on" collision (i.e. b = 0)

\[ d = \text{distance of closest approach} \]

\[ K = \frac{-e^2 Z^2}{4 \pi \varepsilon_0 d} = \frac{\alpha \hbar c Z^2}{d} \]

\[ KE = PE \quad K = \frac{-e^2 Z^2}{4 \pi \varepsilon_0 d} = \frac{\alpha \hbar c Z^2}{d} \]

\[ KE = \text{at } \infty \]

PE at the distance of closest approach

Chapter 7 Hydrogen Atom

You will be given:

\[ \int_0^\infty x^n e^{-cx} \, dx = \frac{n!}{c^{n+1}} \]

You will also be given the \( R_{n\ell}(r) \) for the hydrogen wavefunction:

\[ R_{21} = \frac{1}{\sqrt{3}} \frac{r}{(2a_0)^{3/2}} e^{-r/2a_0} \]

⇒ You should know how to calculate \( \langle r \rangle \) and \( \langle r^2 \rangle \) for various (\( n, \ell \)) states:

For example:

\[ \langle r \rangle_{21} = \int_0^\infty \frac{P(r)}{21} r \, dr = \int \frac{r^2}{(2a_0)^{1/2}} \, dr \]

\[ \langle r \rangle_{21} = \frac{1}{3} \frac{1}{8a_0^3} \frac{1}{a_0^2} \int_0^\infty r^2 e^{-r/2a_0} \, r^3 \, dr = \frac{1}{24a_0^5} \int_0^\infty r^5 e^{-r/2a_0} \, dr \]

\[ \langle r \rangle_{21} = \frac{1}{24a_0^5} \frac{5!}{(1/a_0)^2} = 5a_0 \]
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Angular Momentum in Quantum Mechanics.

You should know \[ l^2 = \frac{l(l+1)}{\hbar} \]
\[ L_z = m_l \hbar \quad \text{where} \quad m_l = 0, \pm 1, \pm 2, \pm 3, \ldots \]

You should know how to calculate the angle \( \Theta \) between \( \vec{L} \) and the \( z \)-axis for a given \( l, m_l \) state:
\[
\cos \Theta = \frac{m_l \hbar}{\sqrt{l(l+1)\hbar}} = \frac{m_l}{\sqrt{l(l+1)}}
\]

Spectroscopic Notation and Selection Rules
\( l = 0, 1, 2, 3, 4, 5, \ldots \)

designation: \( s \quad p \quad d \quad f \quad g \quad h \)

Example: An electron in the hydrogen ground state
\[ \psi_{1,00} \rightarrow 1s \text{ state} \]
\[ \psi_{2,00} \rightarrow 2s \text{ state} \]
\[ \psi_{3,2-1} \rightarrow 3d \text{ state} \quad (m_l \text{ does not play a role in this spectroscopic notation}) \]

It's understood that the 3d state has \( 2l+1 \) \( m_l \) states,
or 5 \( m_l \) states \( \Rightarrow m_l = 0, \pm 1, \text{ and } \pm 2 \).

The most common transitions in atoms are "electric dipole transitions." The selection rule associated with this transition is \[ \Delta l = \pm 1 \quad \left( l_{\text{final}} - l_{\text{initial}} = \pm 1 \right) \left( \text{allowed} \right) \]
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The Zeeman Effect.

Potential Energy due to magnetic dipole in a magnetic field is: \( U = -\vec{\mu}_L \cdot \vec{B} \) where \( \vec{\mu}_L = -\frac{e}{2m} \)

\[
U = -\frac{m_L}{2m} e \hat{L} \cdot \vec{B} = e \frac{L_z B}{2m} = e \frac{(m_e^L) B}{2m}
\]

\[
U = \frac{m_L (e \hbar)}{2m} B \quad U = m_L \mu_B B \]

no magnetic field "Field on"

\( (n, l) = 2p \) state \( l=1 \)

\( \mu_B = \) Bohr magneton \( = 5.788 \times 10^{-5} \text{ eV/T} \)

This constant will be provided.

Example: Using the figure above: \( B = 1.2 \text{ tesla} \)

\( 13.6 \text{ eV} \)

a). Calculate \( E \)

\[
E = -E_R \left( \frac{1}{2^2} - \frac{1}{1^2} \right) = 3 \frac{E_R}{4} = 10.2 \text{ eV}
\]

b). Calculate \( \Delta E \)

\[
\Delta E = \mu_B B = 5.788 \times 10^{-5} \text{ eV/T} (1.2 \text{ T}) = 6.95 \times 10^{-5} \text{ eV}
\]

c). Calculate \( \Delta \lambda \)

\[
\Delta \lambda = \frac{\Delta E}{E} = \frac{1240 \text{ eV nm}}{(10.2 \text{ eV})^2} (6.95 \times 10^{-5} \text{ eV}) = 8.28 \times 10^{-4} \text{ nanometers}
\]

Now, the magnetic quantum number \( m_L \) is visible!!

Selection Rules \( \Delta m_L = 0, \pm 1 \) (for all \( n, l \) states)

\[
|\Delta E| = |\Delta m_L = 1| \mu_B B \quad \Delta E = \mu_B B
\]

\[
\lambda - \Delta \lambda \quad \lambda + \Delta \lambda
\]