Thermal Analysis of a Tungsten Radiation Shield for Beamed Core Antimatter Rocketry

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Recent studies have shown the potential of antimatter propulsion to produce specific impulses on the order of 10^7s. One such concept requires a radiation shield to absorb pure electromagnetic radiation in the form of 511 KeV photons. Different shield materials are analyzed and a thermal analysis is completed on the most probable shield material "tungsten". The required shield size, mass, and maximum rocket thrust is determined as a function of the shield properties.

Nomenclature

A	=	shield Area
c	=	speed of light (3.00 x 10^8 m/s)
c_m	=	specific heat constant
E	=	energy
E_o	=	input energy
e^+	=	positron
e^-	=	electron
F	=	thrust
I_{sp}	=	specific impulse
m	=	rest mass $\left(\sqrt{E}/c\right)$
$\dot{m}$	=	mass flow rate
M_f	=	S/C final mass
M_i	=	S/C initial mass
M_{sh}	=	shield mass
p	=	proton
$\overline{p}$	=	antiproton
p'	=	momentum
Q	=	thermal energy
r_i	=	inner shield radius
r_o	=	outer shield radius
T	=	temperature (K)
V	=	volume
x	=	shield thickness
x_o	=	radiation lengths
γ	=	gamma ray
ε	=	emissivity
υ	=	burnout velocity
ρ	=	density

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Introduction

Pure matter-antimatter annihilation is the most perfect energy source in the known universe. One kilogram of matter and antimatter combined will produce 9.00 x 10^{16}J of energy which can be used to provide momentum (e.g. forward motion) to a spacecraft. Matter-antimatter annihilation provides a fuel specific energy two orders of magnitude greater than the most energetic form of nuclear fusion, DHe^3 fusions. This has made matter-antimatter annihilation the most probable candidate to fuel aggressive manned interplanetary and interstellar space exploration missions.

Matter-antimatter annihilation liberates center-of-mass energy perfectly according the Einstein's famous equation written as:

$$E = \sqrt{m^2c^4 + p^2c^2} \qquad (1)$$

Proton-antiproton annihilation obeys this equation through the formation of massive sub-atomic particles such as pi-mesons, muons, and electron-positron pairs. Proton-antiproton annihilations also produce pure momentum in the formation of high energy photons (gamma rays). Unfortunately $p\,\overline{p}$

rocket engines must efficiently convert pure momentum and kinetic energy (e.g. massive particles with high velocities) into propulsive energy. Attempts have been made to run the resultant massive particles through a working fluid such as liquid Hydrogen to regain the energy from the annihilation, but this has suffered in poor energy transfer to the fluid. In most cases the engines loose the momentum from the pure momentum particles (e.g. photons). Currently $p\bar{p}$ annihilation shows more promise as a driver for nuclear fission and fusion propulsion rather than in pure beamed core antimatter rocketry techniques.[1-2]

Electron-positron annihilation is also another form of matter antimatter annihilation which offers some hope for propulsive applications. When an electron and positron meet, the center of mass energy is liberated completely in the form of momentum. Ninety-nine percent of all electron-positron annihilations produce two _-rays of 511 KeV per photon. The photons are created isotropically and are produced back to back. The annihilation process is written as:

$$e^{+} + e^{-} \rightarrow \gamma\gamma \qquad (2)$$

Theory of Electron Positron Propulsion

A spacecraft equipped with an $e^{+}e^{-}$ engine will annihilate it's fuel to produce gamma rays that will be captured into a momentum transfer shield at the extreme end of a rocket system. Two cases have been considered to capture momentum from the resultant photons as shown in Figure 1.[3]

In the first case $e^{+}e^{-}$ beams are injected into a hemispherical dish that extends from $\pi/2 \rightarrow -\pi/2$, and directed towards a collision at the center of the dish. At this point the $e^{+}e^{-}$ pairs annihilate producing two 511 KeV γ-rays back to back. Half the resultant photons collide with the perfectly absorbing shield, while the other half are ejected into space from the annihilation point. The annihilation process is isotropic which produces a uniform distribution of photon collisions across the dish.

Each collision with the shield causes a transfer of momentum to the dish boosting the spacecraft from one velocity to a higher velocity. This process yields an I_{sp} of (~9.75×10^{6}s), assuming ideal conditions.

In reality the incident photons would cause a scattering effect inside the shield. The photons will collide with atoms of the shield material causing a spray of electrons and lower energy photons to be ejected at random angles. Some of these photons and electrons will be ejected at angles opposite to the direction of motion causing some momentum to be lost. This will serve to decrease the specific impulse and thrust. Currently Monte Carlo simulations are being developed to determine the efficiency lost due to the scattering of electrons and photons.

The second configuration involves the use of a parabolic shield that extends from 5π/6 to -5π/6. In this case the shield is assumed to be a perfect reflector of the 511 KeV photons. This reflection process creates a large increase in the momentum vector of the incident photons. This concept yields a much larger I_{sp} and thrust than with the previous case. Although this is by far the most efficient of the two cases current technology lacks the ability to reflect 511 KeV photons. The I_{sp} from this case is (~3.06×10^{7}s).

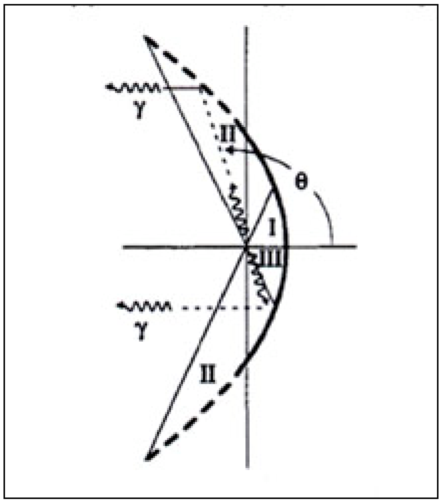

Figure 1. Photon absorbing and reflecting shields. Dashed line represents reflecting shield.

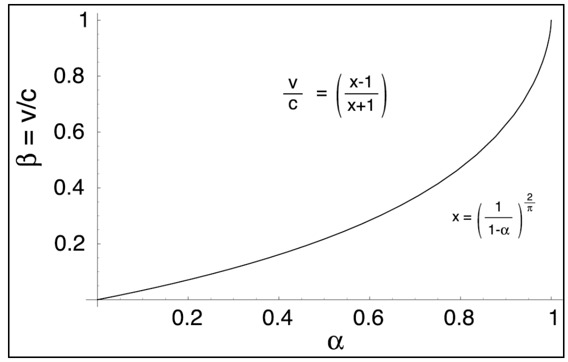

Figure 2. Velocity profile of the absorbing shield concept. _ is the fraction of the spacecrafts fuel used as propellant, _ is the burnout velocity as a fraction of the speed of light, <cos _> = 2/π.

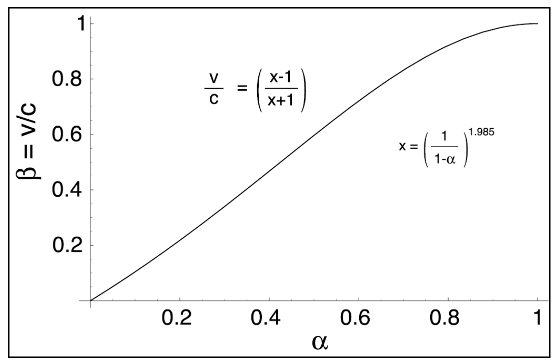

Figure 3. Velocity profile of the reflecting shield concept. _ is the fraction of the spacecrafts mass used as propellant, _ is the burnout velocity as a fraction of the speed of light, <cos _> = 1.985.

A detailed velocity profile of these two configurations has shown incredible burnout velocities for relatively small amounts of fuel (i.e. electrons and positrons) Figure 2,3.[3] The velocity profile graphs are derived from the equation.

$$v = c\left(\frac{1-x}{1+x}\right) \qquad (3)$$

where

$$x = \left(\frac{M_i}{M_f}\right)^{\langle \cos\theta \rangle} = \left(\frac{1}{1-\alpha}\right)^{\langle \cos\theta \rangle} \qquad (4)$$

and α is a running parameter M_p / M_i used to graph the function, $\langle \cos\theta \rangle$ is the mean angular distribution of the momentum that is transferred to the dish in the forward direction by the _-rays. In the case of the absorbing shield the average momentum capture is 2/π, and is 1.985 for the reflecting shield.

Thrust and specific impulse for an electron-positron engine is written as

$$F = \frac{\langle \cos\theta \rangle}{2}(\dot{m}c) \qquad (5)$$

$$I_{sp} = \frac{\langle \cos\theta \rangle c}{2g} \qquad (6)$$

Equations three and four are derived in further detail in a previous study at Embry Riddle Aeronautical University[3].

Shield Selection

Due to the technical problems associated with reflecting 511 KeV photons, near term propulsive applications of this type of rocketry will use the absorbing shield concept. Before a rocket can be built for a test configuration one must know the mass and dimension of the shield required. In this case we are considering a shield for use in a test configuration where 10^{15} positrons are annihilated with an equal amount of electrons producing 180 J of electromagnetic energy and 10^{-7} N.s of impulse.[4]

One must first consider the minimum inner shield area which can sustain the temperature increase without melting. The change in temperature for various materials can simply be found through equation 7 and is shown to be small for 180 J of energy absorption.

$$\Delta T = \frac{Q}{m\,c_m} \qquad (7)$$

The small ΔT of various materials relative to 180 J of input energy allows a designer to arbitrarily select the inner radius of the shield. In this case a radius of 1.2 cm is selected. One of the most important characteristics of any rocket is lightweight components. Hence the shield must be able to efficiently absorb all of the photons while at the same time weigh as little as possible. The thickness and weight of the shield are controlled by the equation of radiation lengths, written as

$$E = E_0\, e^{-x/x_0} \qquad (8)$$

Using eq. 8 as a base parameter with a shield of 1.2 cm inner radius the mass of the various shields as a function of photon energy absorbed is shown in figure 4. Lead, Tungsten and Platinum were selected as shield material candidates due to the combination of their small radiation lengths and relatively low density. Although Platinum does weigh less than the other two materials, it is more expensive and is not as useful in flight applications due to its low emisivity and melting point. As a result Tungsten was chosen as the radiation shield material for the test and flight configurations.

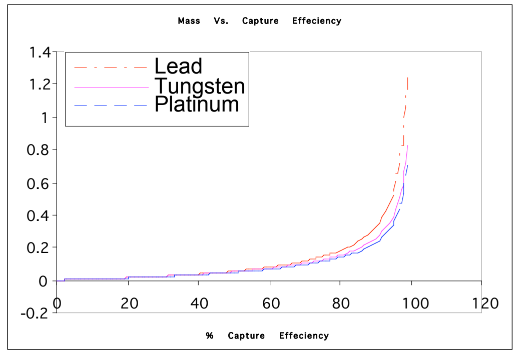

Figure 4. Weight of various shield materials vs. absorbed photon energy for a thruster of 1.2 cm inner diameter.

Thermal Analysis

The thermal properties of the Tungsten shield place practical limitations on the propulsion system. The shield can not be heated past its melting point by the thrusting gamma rays. This places a limit on the maximum amount of propellant which can be injected into the hemisphere at any one time. A constant shield temperature limits the propellant mass flow rate and thrust to a maximum level. The only option which exists to augment the propellant mass flow rate is to increase the surface area of the shield which the photons are incident upon.

Tungsten melts at 3600 K and has an emisivity that ranges from 0.032 at 300 K to 0.354 at its melting point. The maximum steady state thrust at which the shield can cool itself by radiating photons at an equal power density to the incoming radiation can be found through the equation:[5]

$$\dot{m} = \frac{2A\sigma\varepsilon T^4}{c^2} \qquad (9)$$

Only half the photons ejected from the annihilation point will impact the shield thereby raising the temperature. In order to account for this the mass flow rate found in eq. 9 must be doubled and used in eq. 5 to find the steady state thrust at which the shield can operate without melting. A cosine average of $2/\pi$ is assumed in this case.

In this case the shield is assumed to have already been raised to its highest emissivity of 0.354 and temperature of 3600 K by previous electron positron annihilations. Eq. 9 can also be used to relate the mass of the shield as a function of shield area. The equation which relates shield area and mass is given by relating volume and density to area. Mass is related to density through the equation:

$$M_{sh} = \rho_{sh} V \quad (10)$$

The radius required for a hemisphere to have a specified area is given by the equation:

$$r_i = \sqrt{\frac{A_i}{4\pi}} \quad (11)$$

The use of Boolean algebra to subtract the inner radius from the outer radius of the half hemisphere shield is convenient in finding the total volume of the shield:

$$V = \frac{4}{6}\pi r_o^3 - \frac{4}{6}\pi r_i^3 \quad (11)$$

where

$$r_o = r_i + x_o \quad (12)$$

equation 11 can be re written as

$$V = \left[\frac{4}{6}\pi\left(\sqrt{\frac{A_i}{4\pi}} + x_o\right)^3 - \frac{4}{6}\pi\left(\sqrt{\frac{A_i}{4\pi}}\right)^3\right]$$

(13)

equation 13 can then be combined with equation 10 to find the shield mass.

$$M_{sh} = \rho\left[\frac{4}{6}\pi\left(\sqrt{\frac{A_i}{4\pi}} + x_o\right)^3 - \frac{4}{6}\pi\left(\sqrt{\frac{A_i}{4\pi}}\right)^3\right]$$

(14)

The density of tungsten is 19250 kg/m^3. Equation 14 can be used in conjunction with the shield area used in equation 9 and 5 to graph the maximum sustainable thrust and determine the required shield mass at that thrust level, this is shown in figure 5.

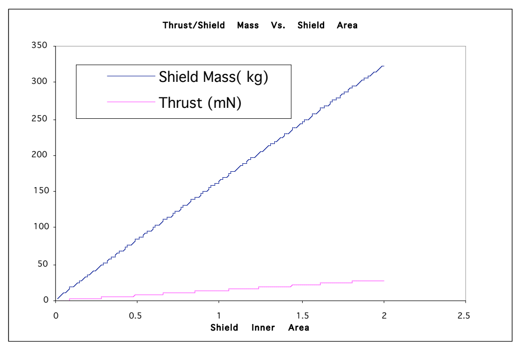

Figure 5. Maximum steady state thrust and shield mass vs. shield inner area.

Figure 5 shown above demonstrates that the sustainable thrust is linearly proportional to the shield area and mass at low thrusts. A maximum of 14 mN of thrust can be maintained with a shield area of 1 m^2 and mass of 165 kg. This corresponds to an energy absorbtion of 13.4 MJ. It should be noted that the emissivity of Tungsten continues to increase with temperature past its melting point. To reach higher thrusts one might allow the Tungsten to pass its melting point and enter the liquid phase in order to reach thrust levels in the higher milli-newtons range. The one draw back is containing the liquid Tungsten in a discernable area. The use of a Tungsten-Uranium Oxide or a Tungsten Carbide alloy in a liquid state also offers to increase the sustainable thrust level towards hundreds of milli-newtons range. A Tungsten-Uranium Oxide shield has an emissivity of 0.79 at a temperature of 1229 K compared to the emissivity of .158 for pure Tungsten. Tungsten Carbide has a boiling boint of 6270 K compared to Tungsten's 5900 K. One should expect that the melting point of Tungsten-Uranium Oxide will decrease from that of pure Tungsten and will also require larger radiation lengths.

A material must be used that not only has a high emissivity, but the material must also have a high melting and boiling point. As long as the emissivity is greater than 0.1 a high melting point will do more to increase the sustainable thrust level than an increase emissivity. A shield material must be able to withstand extremely high temperatures in order to allow for high thrusts as shown in figure 6 below. In order to sustain just one Newton of thrust the material must be able to withstand temperatures of 10,400 K. This is well within the plasma region and would require electromagnets to confine the Tungsten plasma to the shield.

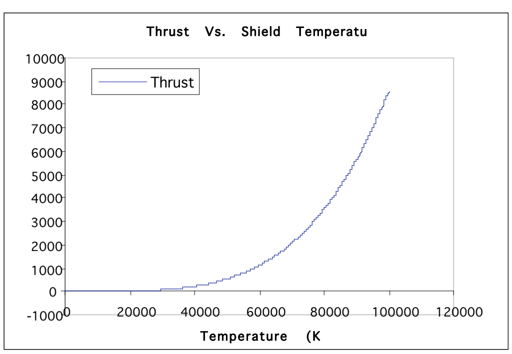

Figure 6. Thrust vs. Shield temperature for a 1 square meter Tungsten shield and an emissivity of 0.2.

Although the use of plasma as a shield is challenging, it is an interesting concept. In a plasma state, Tungsten will have its highest emissivity, and will not necessarily require a shield size as large as $1 m^2$.

The other option for increasing the thrust level involves the use of a shield coolant such as liquid Hydrogen or water. The shield can be regeneratively cooled (e.g. conductively cooled with a liquid). This will allow for larger thrust from the incident photons and from the expanding coolant fluid. The draw back is that this will decrease the specific impulse of the rocket.

Thruster Cooling

By combining eq. 7 and 9 the cooling time for a shield can be found. The equation for cooling time as a function of ΔT is written as:

$$t = \frac{\Delta T m c_m}{A\left[\varepsilon \sigma \left(T_o^4 - T_f^4\right)\right]} \qquad (15)$$

Assuming the initial temperature is 3600 K the cooling time can be graphed as a function of final temperature. The specific heat constant of Tungsten is 132.6 J/kg K and the emmisivity is assumed to be a constant at 0.22. In reality the emmisivity will decrease with temperature. Assuming a lower emmisivity of 0.22 displays a more realistic cooling time for all temperatures. Using the previous graphs a shield area is assumed to be $0.4 m^2$ with a mass of 8 kg, as will be shown later in this paper this is a shield design competitive with other propulsion systems.

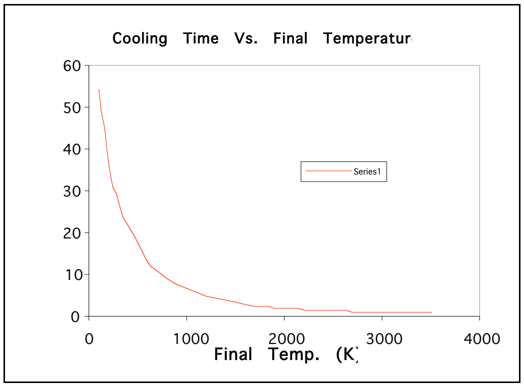

Figure 7. Cooling time for a Tungsten shield vs. final temperature. (assuming initial temperature of 3600 K)

Thruster Analysis

Recently the NASA Glenn Research Center completed testing on the Lincoln Experimental Satellite Thruster (LES-8/9). The LES-8/9 thruster is a pulsed plasma thruster with an approximate mass of 7.5 kg with an average thrust of 573 _N at an I_{sp} of ~1000s.[6] This thruster is used to complete station-keeping maneuvers and attitude control burns on earth orbiting satellites.

Analysis has shown that an absorbing shield antimatter rocket using a pure solid Tungsten shield can maintain the same thrust at an I_{sp} of 9.75 x 10^6 seconds for a thruster mass of 8 kg. This mass does not include the mass of the propellant storage tanks. Although the mass of the antimatter thruster is more than that of the pulsed plasma thruster the propellant consumption is 9750 times less than that of the pulsed plasma thruster. The use of a liquid Tungsten, liquid Tungsten-Uranium Oxide or liquid Tungsten-Carbide shield may allow an antimatter thruster to maintain thrusts of 573 _N at an I_{sp} of 9.75 x 10^6 while having a smaller thruster. Various Tungsten alloy shields may potentially may allow high milli-newton thrust levels for a shield that ways under 50 kg. The possibility of using Tungsten plasma shields may allow multi Newton thrust levels, but this technique has obvious technical challenges associated with it.

Conclusion

The use of a absorbing shield beamed core antimatter rocket for micro-propulsion applications does seem feasible from a thermal viewpoint. Further research must be conducted into the use of a liquid Tungsten shield and various other Tungsten alloys as the upper limits of shield technology for this rocket. Unfortunately the thrust levels that can be reached by the simpler shield configurations are to low for manned spacecraft applications. Ultimately to reach the maximum thrust, impulse, and specific impulse from antimatter rocketry the reflecting shield scenario must be utilized. The reflecting shield scenario has minimal thermal constraints, and does not need a massive radiation shield to maintain thrust.
Due to the difficulty in reflecting 511 KeV photons, the absorbing shield scenario must be used until breakthroughs occur in gamma ray mirror technology.

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