PS113 Chapter 3

Kinematics in two dimensions

1 Displacement, velocity, and acceleration

- From the previous chapter we wrote the displacement vector in one dimension as $\mathbf{\Delta x} = \mathbf{x} - \mathbf{x_o}$. In two dimensions we write the displacement vector as:

$$\mathbf{\Delta r} = \mathbf{r} - \mathbf{r_o}$$

- Likewise, the average velocity vector in two dimensions can be written as:

$$\overline{\mathbf{v}} = \frac{\mathbf{r} - \mathbf{r_o}}{t - t_o} = \frac{\mathbf{\Delta r}}{\Delta t}$$

- Furthermore, the average acceleration in two dimensions can be written as:

$$\overline{\mathbf{a}} = \frac{\mathbf{v} - \mathbf{v_o}}{t - t_o} = \frac{\mathbf{\Delta v}}{\Delta t}$$

Problem 4: A baseball player hits a triple and ends up on third base. A baseball "diamond" is a square, each side of length 27.4 m, with home plate and the three bases on the four corners. What is the magnitude of his displacement?
Answer: 27.4 m

2 Equations of kinematics in two dimensions

- The equations of motion in the x direction are:

$$v_x = v_{ox} + a_x t \tag{1}$$
$$x = \frac{1}{2}\left(v_{ox} + v_x\right) t \tag{2}$$
$$x = v_{ox} t + \frac{1}{2} a_x t^2 \tag{3}$$
$$v_x^2 = v_{ox}^2 + 2 a_x x \tag{4}$$

- Likewise, the equations of motion in the y direction become:

$$v_y = v_{oy} + a_y t \tag{5}$$
$$y = \frac{1}{2}\left(v_{oy} + v_y\right) t \tag{6}$$
$$y = v_{oy} t + \frac{1}{2} a_y t^2 \tag{7}$$
$$v_y^2 = v_{oy}^2 + 2 a_y y \tag{8}$$

- It is important to recognize that the x part of the motion occurs exactly as it would even if the y part did not occur at all. Similarly, the y part of the motion occurs exactly as it would if the x part of the motion did not exist.

- **N.B.** While the motions in the x and y direction are independent of each other, they still share one important kinematical variable, the time t.

Problem 12: A spacecraft is traveling with a velocity of $v_{ox} = 5480$ m/s along the $+x$ direction. Two engines are turned on for a time of 842 s. One engine gives the spacecraft an acceleration in the $+x$ direction of $a_x = 1.20$ m/s^2, while the other gives it an acceleration in the $+y$ direction of $a_y = 8.40$ m/s^2. At the end of the firing, find (a) v_x and (b) v_y.
Answers: $v_x = 6490$ m/s $\quad$ $v_y = 7073$ m/s

3 Projectile Motion

- In our study of projectile motion, we can use the 8 equations from the previous section and substitute $a_x = 0$, and $a_y =$ -9.8 m/s^2 (assuming there's no air resistance).
- The time a projectile is in flight is usually obtained from solving the equations of motion in the y direction.
- The range R that a projectile travels in the x direction is obtained from the following equation:

  $$R = v_{ox}t$$

  In some cases it may appear that only two of the five kinematical quantities are known in the x direction; however, the time t can usually be obtained from the kinematical quantities known in the y direction.

Problem 18: A horizontal rifle is fired at a bull's-eye. The muzzle speed of the bullet is 670 m/s. The barrel is pointed directly at the center of the bull's-eye, but the bullet strikes the target 0.025 m below the center. What is the horizontal distance between the end of the rifle and the bull's-eye?
Answer: 47.9 m

Problem 27: A fire hose ejects a stream of water at an angle of 35.0^o above the horizontal. The water leaves the nozzle with a speed of 25.0 m/s. Assuming that the water behaves like a projectile, how far from a building should the fire hose be locatd to hit the highest possible fire?
Answer: 30.0 m

4 Relative Velocity

In order to determine the velocity between inertial frames (i.e., constant velocity frames), we can use vectors *with some subscripts* to simplify some of the tedious calucations that arise in 2 and 3 dimensions. This is best illustrated in a familiar example.

- Suppose we know the air speed of an airplane: $\vec{v}_{PA}$ the velocity of the plane *with respect to* the air.

- Suppose we know the wind speed: $\vec{v}_{AG}$ the velocity of the air *with respect to* the ground.

- Then, we can calculate the ground speed of an airplane: $\vec{v}_{PG}$ the velocity of the plane *with respect to* the ground.

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG} \tag{9}$$

Problem 47: The drawing shows an exaggerated view of a rifle that has been "sighted in" for a 91.4-meter target. If the muzzle speed of the bullet is $v_o = 427$ m/s, what are the two possible angles θ_1 and θ_2 between the rifle barrel and the horizontal such that the bullet will hit the target? One of these angles is so large that it is never used in target shooting *(Hint: The following trigonometric identity may be useful:* $2\sin\theta\cos\theta = \sin 2\theta$*.)*