Practice Exam #2 **PS250**
June 19, 2006 Dr. Darrel Smith

Show your work !!

Answer Key
Name

10 points

1. A strand of wire has resistance 5.60 μΩ. Find the net resistance of 120 such strands if they are:

   a) placed side by side to form a cable of the same length as a single strand.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{120}} = \frac{N}{R} \qquad R_{eq} = \frac{R}{N} = \frac{5.60\mu\Omega}{120} = 4.67\times10^{-8}\Omega$$

R = 4.67×10^{-8} Ω

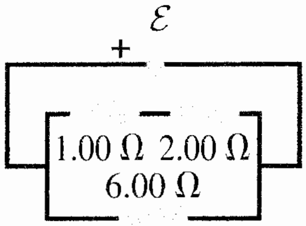
   b) connected end to end to form a wire 120 times as long as a single strand.

$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_{120} = NR = 120(5.60\mu\Omega) = 6.72\times10^{-4}\Omega$$

R = 6.72×10^{-4} Ω

15 points

2. A resistor circuit is shown in the figure to the right. The voltage across the 2.00Ω is 12.0 V.



   a. What is the emf of the battery?

$$i_{2\Omega} = \frac{V}{R_{2\Omega}} = \frac{12\,V}{2\Omega} = 6A \qquad V_{drop}(1\Omega) = IR_{1\Omega} = 6A(1\Omega)$$

$$V_{drop}(1\Omega) = 6 \text{ volts}$$

$$\mathcal{E} = 12V + 6V = 18\ V$$

emf = 18.0 volts

   b. What is the current in the 1.00Ω resistor?

$$i_{1\Omega} = i_{2\Omega} = 6A$$

$I_{1\Omega}$ = 6.00 A

   c. What is the current in the 6.00Ω resistor?

$$i_{6\Omega} = \frac{V}{R_{6\Omega}} = \frac{18.0\,V}{6\Omega} = 3A$$

$I_{6\Omega}$ = 3.00 A

   d. Calculated the power dissipated by each resistor

$$P = I^2R \qquad P_1 = (6A)^2\,1\Omega = 36\ W$$

$$P_2 = (6A)^2\,2\Omega = 72\,W$$

$$P_3 = (3A)^2\,6\Omega = 54\,W$$

$P_{1\Omega}$ = 36 W
$P_{2\Omega}$ = 72 W
$P_{6\Omega}$ = 54 W

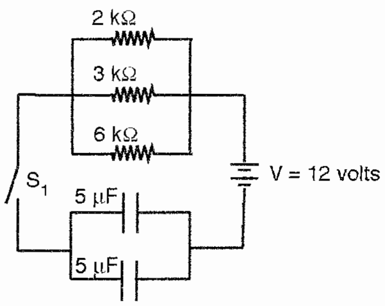
   e. Calculate the power delivered by the battery.

$$I_{battery} = 6A + 3A = 9\,A \qquad P_{battery} = VI = (18\,V)(9\,A)$$

$P_{battery}$ = 162 W

15 points

3. An *R-C* circuit is shown in the figure to the right.



   a. Calculate the time constant for this circuit.

$$\frac{1}{R_{eq}} = \frac{1}{2k\Omega} + \frac{1}{3k\Omega} + \frac{1}{6k\Omega} = \frac{6}{6k\Omega} \Rightarrow R_{eq} = 1\,k\Omega$$

$$C_{eq} = 5\mu F + 5\mu F = 10\mu F \qquad \tau = R_{eq}C_{eq} = (1\times10^3\Omega)(10\times10^{-6}F)$$

$$\underline{\tau = 10\ ms}$$

$\tau =$ 10 ms

b. At what time do the capacitors reach 90% of their final charge?

$$Q = Q_f\left(1-e^{-t/RC}\right) \Rightarrow 0.90\,Q_f = Q_f\left(1-e^{-t/RC}\right)$$

$$0.90 = 1-e^{-t/RC} \Rightarrow e^{-t/RC} = \frac{1}{10} \quad -\frac{t}{RC} = \ln\left(\frac{1}{10}\right) \Rightarrow t = RC\ln(10)$$

$$t = 10\,ms\,\ln(10) = 23.0\,ms$$

t = 23.0 ms

c. What is the instantaneous power delivered by the battery at t = 20 ms?

$$P = VI \quad \text{where } I = I_o e^{-t/RC} \quad \text{where } I_o = \frac{V}{R_{eq}} = 12\,mA$$

$$I = 12\,mA\,e^{-20/10} \quad I = 1.62\,mA \quad P = VI = (12\,V)(1.62\,mA)$$

$P_{battery} =$ 19.5×10^{-3} W

10 points

4. A proton traveling at 2/3 of the speed of light enters a region of homogeneous magnetic field (1.30 T). If the velocity vector is perpendicular to the direction of the magnetic field,

a) Calculate the radius of curvature of the proton.

$$R = \frac{mv}{qB} = \frac{(1.67\times10^{-27}\,kg)\,\frac{2}{3}(3.0\times10^{8}\,m/s)}{(1.602\times10^{-19}\,C)(1.30\,T)} = 1.60\,m$$

R = 1.60 cm

b) Calculate the frequency of the orbiting proton.

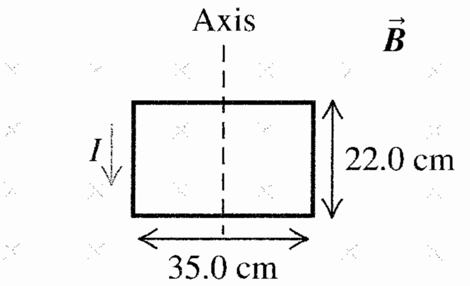
$$v = R\omega \quad \omega = \frac{v}{R} = \frac{v\,qB}{mv} = \frac{qB}{m} \quad \omega = 2\pi f$$

$$f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m} = \frac{(1.602\times10^{-19})(1.30)}{2\pi(1.67\times10^{-27})} = 1.98\times10^{7}\,s^{-1}$$

f = 1.98×10^{7} sec^{-1}

15 points

5. A rectangular coil of wire, 22.0 cm by 35.0 cm and carrying a current of 1.40 A, is oriented with the plane of its loop perpendicular to a uniform 1.50-T magnetic field, as shown in the figure. If the coil is rotated through a 30.0° angle about the axis shown, the left side coming out of the plane of the figure and right side going into the plane,



a. Calculate the magnetic dipole moment for this current loop.

$$\mu = IA = 1.40A\,(35\times22)\times10^{-4}\,m^2 = 0.108\,A\cdot m^2$$

$$\vec{\mu} = (\sin30^\circ\,\hat{i} + \cos30^\circ\,\hat{k})(0.108\,A\cdot m^2)$$

$\mu =$ 0.108 $A\cdot m^2$

b. Calculate the torque vector. (Assume x is to the right and y is up.)

$$\vec{\tau} = \vec{\mu}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \sin30^\circ & 0 & \cos30^\circ \\ 0 & 0 & -1.50 \end{vmatrix}(0.108\,A\cdot m^2) = \left[\hat{i}(0) - \hat{j}\left(\tfrac{1}{2}(-1.50)\right) + \hat{k}(0)\right] \times (0.108\,A\cdot m^2)$$

$$\vec{\tau} = (8.10\times10^{-2}\,N\cdot m)\,\hat{j}$$

$\vec{\tau} =$ 0 $\hat{i}$ + 8.10×10^{-2} $\hat{j}$ + 0 $\hat{k}$ N·m

c. Calculate the potential energy

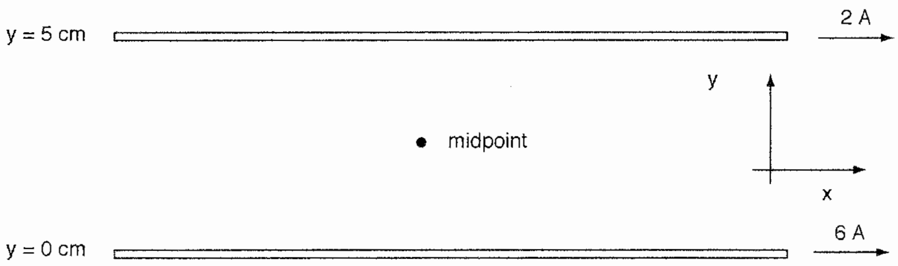
$$U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos30^\circ = -(0.108\,A\cdot m^2)(1.50\,T)\cos30^\circ$$

$$U = -0.140\,J$$

$U =$ -0.140 J

15 points

6. Two infinitely long currents (shown in the figure below) are parallel and separated by 5.00 cm in a vacuum.



a. Calculate the magnetic field at a midpoint between the two wires.

$$B = \frac{\mu_0 I}{2\pi r} \qquad \vec{B}_{2A} = \frac{\mu_0 (2A)}{2\pi (0.025)}(-\hat{k}) = \frac{4\pi \times 10^{-7}(2A)}{2\pi (0.025)}(-\hat{k}) = (-1.60 \times 10^{-5}\ T)\hat{k}$$

$$\vec{B}_{6A} = \frac{\mu_0 (6A)}{2\pi (0.025)}(\hat{k}) = \frac{4\pi \times 10^{-7}}{2\pi}\ \frac{6A}{0.025}\ \hat{k} = (+4.80 \times 10^{-5}\ T)\hat{k} \qquad \boxed{3.20 \times 10^{-5}\ T}$$

$\vec{B}$ = (0 $\hat{i}$ + 0 $\hat{j}$ + 3.20×10^{-5} $\hat{k}$) T

b. At what location in y is the magnetic field equal to zero?

$$B_{6A} = \frac{\mu_0 I_{6A}}{2\pi y} \qquad B_{2A} = \frac{\mu_0 I_{2A}}{2\pi (5-y)} \qquad B_{TOTAL} = B_{6A} - B_{2A} = 0$$

$$\frac{\mu_0 6A}{2\pi y} - \frac{\mu_0 2A}{2\pi (5-y)} = 0 \qquad \frac{6}{y} - \frac{2}{5-y} = 0 \nearrow \frac{y}{3} = 5 - y \rightarrow \frac{4}{3}y = 5 \quad y = \frac{15}{4}\text{ cm}$$

$y =$ 3.75 cm

c. Calculate the *force per unit length* between the two current-carrying wires.

$$\frac{F}{L} = \frac{\mu_0 I_{2A} I_{6A}}{2\pi r} = \frac{\mu_0 (2A)(6A)}{2\pi (0.05)} = \frac{4\pi \times 10^{-7}}{2\pi}\ \frac{12}{.05} = \boxed{4.80 \times 10^{-5}\ \frac{N}{m}}$$

d. Is the force *attractive* or *repulsive* ? (circle one)

$$\vec{F} = I\ \vec{L} \times \vec{B}$$