Practice Exam PS250
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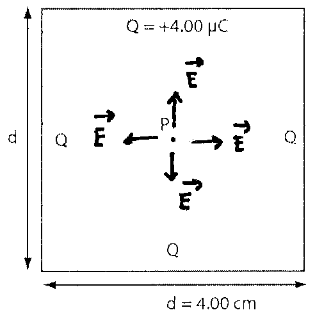
**Show your work !!**

Answer Key
Name

**10 points**

1. A charge Q is uniformly distributed on each of four equal lengths as shown in the figure to the right. If Q = 4.00 μC and d = 4.00 cm,




a. Calculate the magnitude of the electric field at the center (P) of the square.

By symmetry, the electric field = 0

E = ________ N/C

b. Calculate the potential at the center (P) of the square

$V_o$ = potential for one line segment $= \frac{1}{4\pi\epsilon_o}\frac{Q}{2a}\ln\left(\frac{\sqrt{(d/2)^2+(d/2)^2}+d/2}{\sqrt{(d/2)^2+(d/2)^2}-d/2}\right)$, $a = d/2$

$$V_o = \frac{1}{4\pi\epsilon_o}\frac{Q}{2a}\ln\left(\frac{\sqrt{2}\,d/2 + d/2}{\sqrt{2}\,d/2 - d/2}\right) = \frac{1}{4\pi\epsilon_o}\frac{Q}{d}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) = 1.58\times10^6\text{ V}$$

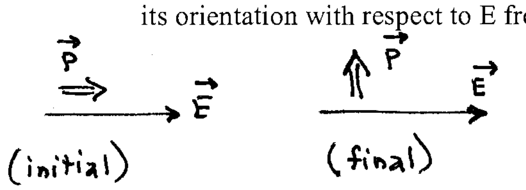
$a = d/2$

$V = 4V_o = 6.34\times10^6$ V

V = $6.34\times10^6$ volts

**10 points**

2. The ammonia molecule ($NH_3$) has a dipole moment of $5.0\times10^{-30}$ C·m. Ammonia molecules in the gas phase are placed in a uniform electric field E with magnitude $1.6\times10^6$ N/C. What is the change in the electric potential energy when the dipole moment of a molecule changes its orientation with respect to E from parallel to perpendicular?




$$\Delta U = -(U_f - U_i) = -(-\vec{P}_f\cdot\vec{E} - (-\vec{P}_i\cdot\vec{E}))$$

$$\Delta U = \vec{P}_f\cdot\vec{E} - \vec{P}_i\cdot\vec{E} = 0 - pE\cos 0^\circ$$

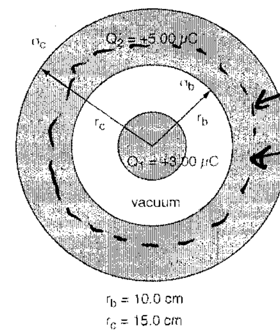
$$\Delta U = -pE = (-5\times10^{-30}\text{ C·m})(1.6\times10^6\text{ N/C})$$

$$\Delta U = -8.00\times10^{-24}\text{ J}$$

ΔU = $-8.00\times10^{24}$ Joules

**10 points**

3. Two conducting spheres are shown in the figure to the right. The innermost sphere of radius 5.00 cm is solid and contains +3.00 μC of charge. The second sphere is concentric having an inner radius of 10.0 cm and outer radius of 15.0 cm while holding +5.00 μC of charge. Find the surface charge densities, $\sigma_b$ and $\sigma_c$.




E = 0

gaussian sphere

$\Phi = \frac{Q_{enc}}{\epsilon_o} = 0$

$Q_b = -3.00\,\mu C$ so E = 0 on the gaussian sphere

$Q_c = 5.00\,\mu C - Q_b = 8.00\,\mu C$

$$\sigma_b = \frac{Q_b}{4\pi r_b^2} = \frac{-3.00\times10^{-6}\text{ C}}{4\pi(0.10\text{ m})^2} = 23.9\,\frac{\mu C}{m^2}$$

$\sigma_b$ = 23.9 μC/m²

$$\sigma_c = \frac{Q_c}{4\pi r_c^2} = \frac{8.00\times10^{-6}\text{ C}}{4\pi(0.15\text{ m})^2} = 28.3\times10^{-6}\,\frac{C}{m^2}$$

$\sigma_c$ = 28.3 μC/m²

**10 points**

4. An electron is accelerated from rest in a uniform electric field between two capacitor plates. If the electron is release from the cathode (the negatively charged plate), and the potential difference between the plates is 12,000 V,

   a. What is the speed of the electron when it strikes the anode (the positively charged capacitor plate? $m_e = 9.11 \times 10^{-31}$ kg

$qV = \frac{1}{2}mv^2 \qquad v^2 = \frac{2qV}{m} \qquad v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2(1.602\times10^{-19})(12{,}000)}{9.11\times10^{-31}}}$

$v = 6.50 \times 10^{7}$ m/s

v = $6.50\times10^{7}$ m/s

   b. If the separation between the plates is 1.20 cm, how long will it take for the electron to travel from the cathode plate to the anode plate?

$V_o = 0$
$V = 6.50\times10^{7}$ m/s
$X = 0.012$ m
$t = ?$

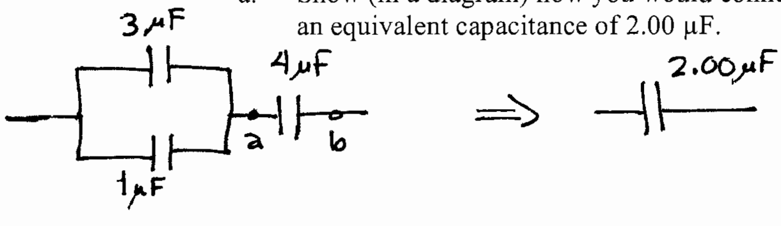
$x = \frac{1}{2}(v_o + v)t \qquad t = \frac{2x}{V + V_o} = \frac{2(0.012\text{ m})}{6.5} = 3.69\times10^{-10}$ s

t = $3.69\times10^{-10}$ s

**20 points**

5. You are handed three capacitors, $C_1 = 3.00$ μF, $C_2 = 1.00$ μF, and $C_3 = 4.00$ μF. However, you need to build a circuit where the equivalent capacitance is 2.00 μF.

   a. Show (in a diagram) how you would connect the capacitors to form a network having an equivalent capacitance of 2.00 μF.




   b. If this circuit is connected to a 12.0 V battery, how much energy must the battery provide to charge up all three capacitors?

$U = \frac{1}{2}CV^2 = \frac{1}{2}(2\times10^{-6}\text{ F})(12\text{ V})^2 = 1.44\times10^{-4}$ J

Energy = $1.44\times10^{-4}$ J

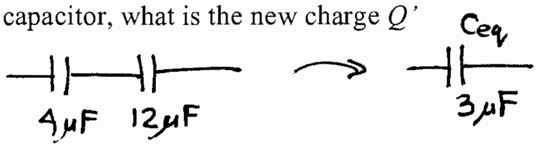
   c. What is the charge $Q$ on the 4.00 μF capacitor?

$V_{ab} = 6.0$ Volt $\qquad Q = VC = (6.0\text{ V})\,4\mu\text{F} = 24\mu\text{C}$

$Q$ = 24.0 μC

   d. If a dielectric ($K$=3.00) is inserted in the 4.00 μF capacitor, what is the new charge $Q'$ on the 4.00 μF capacitor?

Capacitance changes from 4μF → 12μF




$Q' = V C_{eq} = 12\text{ V}(3\mu\text{F}) = 36\mu\text{C}$

$Q'$ = 36 μC

**Useful constants:** $k = 8.988 \times 10^9$ N·m²/C² $\qquad \varepsilon_o = 8.85 \times 10^{-12}$ C²/N·m²
$e = 1.602 \times 10^{-19}$ C