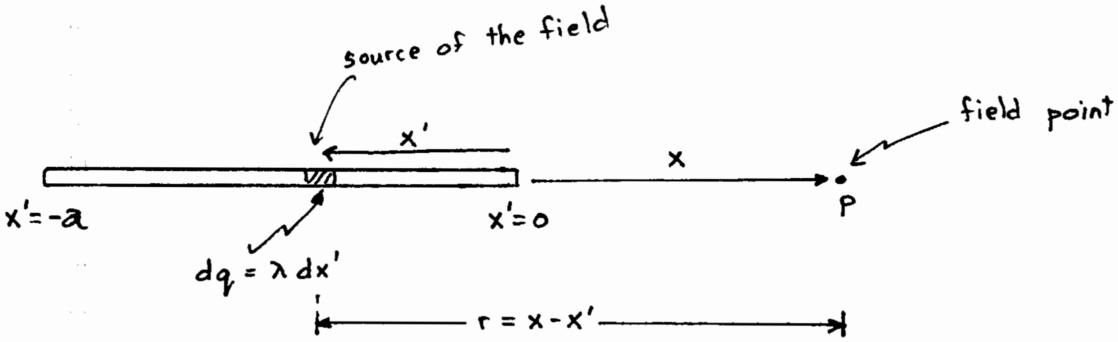
79.

$$\lambda = \frac{Q}{a} \qquad V = \int \frac{k\,dq}{r} = k\int_{-a}^{0} \frac{\lambda\,dx'}{x-x'} = k\lambda \int_{-a}^{0} \frac{dx'}{x-x'} = -k\lambda \ln(x-x')\Big|_{-a}^{0}$$



$$V = -k\lambda\left(\ln x - \ln(x+a)\right) = -k\lambda \ln\left(\frac{x}{x+a}\right) = k\lambda \ln\left(\frac{x+a}{x}\right) = k\lambda \ln\left(1+\frac{a}{x}\right)$$

$$\boxed{V(x) = \frac{Q}{4\pi\varepsilon_0 a} \ln\left(1+\frac{a}{x}\right)}$$

Now use the expansion $\ln(1+z) \simeq z - \frac{z^2}{2} + \cdots$

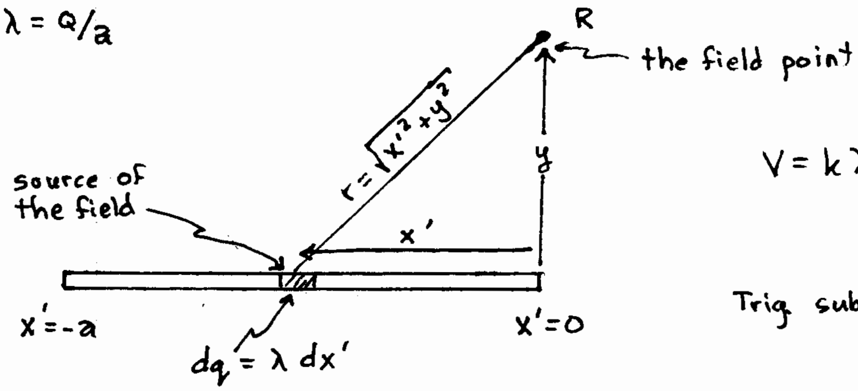
$$V(x) \cong \frac{Q}{4\pi\varepsilon_0 a}\left(\frac{a}{x} - \frac{1}{2}\frac{a^2}{x^2} + \cdots\right)$$

If we choose $x \gg a$, then the 2nd term $\left(\frac{1}{2}a^2/x^2\right)$ is much much smaller than the 1st term (a/x) and you can ignore its contribution.

$$\boxed{V(x) \cong \frac{Q}{4\pi\varepsilon_0 x}}$$ when $x \gg a$.

Conclusion: When the field point is a long distance away from the finite line charge of length a, it appears to generate an electrostatic potential similar to a point charge (Q) located at $x'=0$.

$\lambda = Q/a$



$$V = \int \frac{k\,dq}{r} = k\int_{-a}^{0} \frac{\lambda\,dx'}{\sqrt{x'^2+y^2}}$$

$$V = k\lambda \int_{-a}^{0} \frac{dx'}{\sqrt{x'^2+y^2}}$$

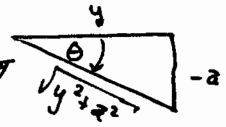
Trig substitution: $x' = y\tan\theta$

$dx' = y\sec^2\theta\, d\theta$

79 cont'd

$$V = k\lambda \int_{\theta=\tan^{-1}\left(\frac{-a}{y}\right)}^{\theta=0} \frac{y \sec^2\theta \, d\theta}{\sqrt{y\tan^2\theta + y^2}} = k\lambda \int_{\tan^{-1}\left(\frac{-a}{y}\right)}^{0} \frac{y \sec^2\theta \, d\theta}{\sqrt{y^2(\tan^2\theta + 1)}} = k\lambda \int_{\tan^{-1}\left(\frac{-a}{y}\right)}^{0} \frac{\sec^2\theta}{\sec\theta}\, d\theta$$

$$V = k\lambda \int_{\tan^{-1}\left(\frac{-a}{y}\right)}^{0} \sec\theta \, d\theta = k\lambda \ln\left|\sec\theta + \tan\theta\right| \Bigg|_{\tan^{-1}\left(\frac{-a}{y}\right)}^{0}$$



$$V = k\lambda \left[\ln\left|\sec 0 + \tan 0\right| - \ln\left|\frac{\sqrt{y^2+a^2}}{y} - \frac{a}{y}\right|\right]$$

$$V = k\lambda\left[\ln|1+0| - \ln\left(\frac{\sqrt{y^2+a^2} - a}{y}\right)\right] = k\lambda\left[0 - \ln\left(\frac{\sqrt{y^2+a^2} - a}{y}\right)\right]$$

$$V(y) = -k\lambda \ln\left(\frac{\sqrt{y^2+a^2} - a}{y}\right) = -k\lambda \ln\left(\frac{\left(\sqrt{y^2+a^2} - a\right)\left(\sqrt{y^2+a^2} + a\right)}{y \left(\sqrt{y^2+a^2} + a\right)}\right)$$

$$V(y) = -k\lambda \ln\left(\frac{y^2 + a^2 - a^2}{y\left(\sqrt{y^2+a^2} + a\right)}\right) = -k\lambda \ln\left(\frac{y}{\sqrt{y^2+a^2} + a}\right) = k\lambda \ln\left(\frac{\sqrt{y^2+a^2} + a}{y}\right)$$

$$\boxed{V(y) = k\lambda \ln\left(\sqrt{1 + \frac{a^2}{y^2}} + \frac{a}{y}\right)}$$

From the binomial expansion: $\sqrt{1+z} = (1+z)^{1/2} \cong 1 + \frac{1}{2}z + \cdots$

$$V(y) \cong k\lambda \ln\left(1 + \frac{1}{2}\frac{a^2}{y^2} + \cdots + \frac{a}{y}\right) \cong k\lambda \ln\left(1 + \frac{a}{y} + \underbrace{\frac{1}{2}\frac{a^2}{y^2} + \cdots}_{}\right)$$

These terms are small for $y \gg a$.

So, $V(y) \cong k\lambda \ln\left(1 + \frac{a}{y}\right)$

The series expansion for $\ln(1+z) \cong z - \frac{z^2}{2} + \cdots$

$$V(y) \cong k\lambda\left(\frac{a}{y} \underbrace{- \frac{a^2}{2y^2} + \cdots}_{}\right) \cong k\lambda \frac{a}{y} = \frac{kQ}{y}$$

These terms are small for $y \gg a$

$$\boxed{V(y) \cong \frac{Q}{4\pi\varepsilon_0 y}}$$

for $y \gg a$