# Chapter 21 Electric Charge and Electric Field

Electromagnetism is one of the four fundamental forces.

1. Gravity (long range)
2. Electromagnetism (long range)
3. Strong (nuclear force, short range)
4. Weak (short range)

## 1 Electric Charge

1. There are two kinds of charges–*positive* and *negative*
2. Charge is quantized. It comes in units of $e$
   - $q_{proton} = +e$
   - $q_{neutron} = 0\,e$
   - $q_{electron} = -e$
   - $q_{up} = +\frac{2}{3}e$ $\qquad q_{down} = -\frac{1}{3}e$ etc.

where $e = 1.602176462(63) \times 10^{-19}$ Coulombs

**Definitions:**

- **Atomic Number** of an element–the number of protons in the nucleus
- **Positive Ion**–the result of removing one or more electrons from a neutral atom
- **Negative Ion**–a previously neutral atom that has acquired one or more electrons
- **Ionization**–the act of gaining or losing electrons from an atom.

**Conservation of Charge** The algebraic sum of all the electric charges in any closed system is constant.

## 2 Conductors, Insulators, and Induced Charges

- Conductors–electrons are mobile and can easily move
- Insulators–do not permit the easy movement of charge through them

**There are two kinds of charges**–positive (+) and negative(-).

Demonstration

Induction–an object can become polarized (i.e., charges move to opposite extremes of the object) without the transfer of charge.

Demonstration

## 3 Coulomb's Law

Charles Augstin de Coulomb (1736-1806) studied the interaction forces due to charged particles using a torsion balance. He discovered that the force between two point charges $q_1$ and $q_2$ is proportional the produce of the two charges.

**Coulomb's Law:** *The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.*

$$F = k\,\frac{|q_1\,q_2|}{r^2} \tag{1}$$

The force is *attractive* if the sign of the charges are opposite. Likewise, the force is *repulsive* if the sign of the charges are the same.

The SI unit of electric charge is called the **coulomb** (C). The constant $k$ is

$$k = 8.987551787 \times 10^9 \, \text{N}\cdot\text{m}^2/\text{C}^2 \approx 8.988 \times 10^9 \, \text{N}\cdot\text{m}^2/\text{C}^2$$

The constant of proportionality $k$ can also be written the following way:

$$k = \frac{1}{4\pi\epsilon_o}$$

where $\epsilon_o$ is the electric permittivity of a vacuum.

$$\epsilon_o \ = \ 8.854 \times 10^{-12} \ \text{C}^2/(\text{N}\cdot \, \text{m}^2)$$

**Ex. 1** Excess electrons are placed on a small lead sphere with mass 8.00 g so that its net charge is $-3.20 \times 10^{-9}$ C. a) Find the number of excess electrons on the sphere. b) How many excess electrons are there per lead atom? The atomic number of lead is 82, and its atomic mass is 207 g/mol.

**Ex. 11** Three point charges are arranged on a line. Charge $q_1 = +5.00$ nC and is at the origin. Charge $q_2 = -3.00$ nC and is at $x = +4.00$ cm. Charge $q_1$ is at $x = +2.00$ cm. What is $q_1$ (magnitude and sign) if the net force on $q_3$ is zero?

## 4 Electric Field and Electric Forces

The electric force is an *action at a distance* force, similar to gravity. When describing the force between two charges, nothing is physically (i.e., visibly) pushing or pulling on the two charges. In these kinds of situations, we develop the concept of a field to "explain" how the charges exert their force on each other.

**Note:** we can also do the same with gravitational force.

How can we describe the force on one charge $q_o$ due to another charge $q$ using this *field* concept?

$$F = k\,\frac{|q\,q_o|}{r^2}$$

Let's identify the electric force on $q_o$ due to $q$ as $F_o$. Then we can define the **electric field** produced by $q$ as

$$\vec{E} \;=\; \frac{\vec{F}}{q_o} \;=\; \lim_{q_o \to 0} \frac{\vec{F}_o}{q_o} \;=\; k\,\frac{q}{r^2}\,\hat{\mathbf{r}} \qquad \text{(electric field of a point charge)}$$

where $\hat{\mathbf{r}}$ is the unit vector pointing from the charge $q$ to the *field point* P.

The force on any test charge $q_o$ can simply be written as:

$$\vec{F} \;=\; q_o\,\vec{E}$$

where $\vec{\mathbf{E}}$ is the electric field. The electric field is a **vector field** and can be written as:

$$\vec{\mathbf{E}}(x, y, z) \;=\; E_x(x, y, z)\hat{\imath} + E_y(x, y, z)\hat{\jmath} + E_z(x, y, z)\hat{k}$$

**N.B.** The electric field is a property of space $\vec{\mathbf{E}}(x, y, z)$.

In some situations the magnitude and direction of the field have the same values everywhere throughout a certain region of space; we say that the electric field is *uniform* in this region.

**N.B.** If a conductor is placed in an electric field, the charges will migrate until they rearrange themselves in such a way as to create a *zero* electric field within the conductor; at which point, the remaining valence electrons are no longer influenced to move.

**Example** Motion of an electron in a uniform electric field

1. Parallel to the electric field
2. Perpendicular to the electric field

**Ex. 25** An alpha particle (charge $+2e$ and mass $6.64 \times 10^{-27}$ kg) is traveling to the right at 1.50 km/s. What uniform electric field (magnitude and direction) is needed to cause it to travel to the left at the same speed after 2.65 $\mu$s?

**Ex. 31** An electron is projected with an initial speed $v_o = 1.60 \times 10^6$ m/s into the uniform field between the parallel plates in Fig. 21.32. Assume that the field between the plates is uniform and directed vertically downward, and that the field outside the plates is zero. The electron enters the field at a point midway between the plates. a) If the electron just misses the upper plate as it emerges from the field, find the magnitude of the electric field. b) Suppose that in Fig. 21.32 the electron is replaced by a proton with the same initial speed $v_o$. Would the proton hit one of the plates? If the proton would not hit one of the plates, what would be the magnitude and direction of its vertical displacement as it exits the region between the plates? c) Compare the paths traveled by the electron and the proton and explain the differences. d) Discuss whether it is reasonable to ignore the effects of gravity for each particle.

## 5 Electric-Field Calculations

The real utility of using the electric field becomes apparent when dealing with multiple-charge systems. In this case, we can use the principle of *superposition* to replace the multiple electric fields with just "one" electric field. The force on a test charge $q_o$ due to multiple charges $q_1$, $q_2$, $q_3$, ... is:

$$\tilde{\mathbf{F}}_{\mathbf{o}} = \tilde{\mathbf{F}}_{\mathbf{1}} + \tilde{\mathbf{F}}_{\mathbf{2}} + \tilde{\mathbf{F}}_{\mathbf{3}} + \cdots = q_o\tilde{\mathbf{E}}_{\mathbf{1}} + q_o\tilde{\mathbf{E}}_{\mathbf{2}} + q_o\tilde{\mathbf{E}}_{\mathbf{3}} + \cdots = q_o(\tilde{\mathbf{E}}_{\mathbf{1}} + \tilde{\mathbf{E}}_{\mathbf{2}} + \tilde{\mathbf{E}}_{\mathbf{3}} + \cdots)$$

$$\tilde{\mathbf{F}}_{\mathbf{o}} = q_o\,\tilde{\mathbf{E}}$$

where $\tilde{\mathbf{E}}$ is the vector sum (i.e., superposition) of the individual electric fields $\tilde{\mathbf{E}}_{\mathbf{1}}$, $\tilde{\mathbf{E}}_{\mathbf{2}}$, and so on.

**Ex. 40** Two particle having charges $q_1 = 0.500$ nC and $q_2 = 8.00$ nC are separated by a distance of 1.20 m. At what point along the line connecting the two charges is the total electric field due to the two charges equal to zero?

## 5.1 Field of an electric dipole

Equal, but opposite charges $q$ and $-q$ are aligned along the $y$-axis separated by a distance $d$.

$$\tilde{\mathbf{E}}(x,y) = kq\,\frac{(x\hat{\imath} + (y - d/2)\hat{\jmath})}{(x^2 + (y - d/2)^2)^{3/2}} - kq\,\frac{(x\hat{\imath} + (y + d/2)\hat{\jmath})}{(x^2 + (y + d/2)^2)^{3/2}}$$

## 5.2 Field of a ring of charge

Calculate the electric field along the axis of symmetry (i.e., the $x$-axis) for a uniform ring of charge of radius $a$.

$$\vec{\mathbf{E}} = \frac{1}{4\pi\epsilon_o}\frac{Qx}{(x^2 + a^2)^{3/2}}\,\hat{\imath}$$

## 5.3 Field of a line of charge

Calculate the electric field along a bisector ($x$-axis) of a uniform linear charge density $\lambda$ along the $y$-axis of length $2a$.

$$\vec{\mathbf{E}} = \frac{1}{4\pi\epsilon_o}\frac{Q}{x\sqrt{x^2 + a^2}}\,\hat{\imath}$$

If the line of charge is infinitely long ($a \to \infty$), then

$$\vec{\mathbf{E}} = \frac{\lambda}{2\pi\epsilon_o x}\,\hat{\imath}$$

**Ex. 48** A very long, straight wire has charge per unit length $1.50 \times 10^{-10}$ C/m. At what distance from the wire is the electric field magnitude equal to 2.50 N/C.

### 5.4 Field of a uniformly charged disk

Calculate the electric field along the axis of symmetry ($x$-axis) due to a uniformly charged disk of radius $R$.

$$\vec{\mathbf{E}} = \frac{\sigma}{2\epsilon_o}\left[1 - \frac{1}{\sqrt{(R^2/x^2)+1}}\right]\hat{\imath}$$

If the disk is infinitely large ($R \to \infty$), then

$$E = \frac{\sigma}{2\epsilon_o}$$

**Ex. 51** A uniformly charged disk of radius $R$ carries positive charge per unit area $\sigma$, as in Fig. 21.23. For points on the $+x$-axis, graph the x-component of the electric field as a function of $x$ for values of $x$ between $x = 0$ and $x = 4R$.

### 5.5 Field of two oppositely charged infinite sheets

$$\vec{\mathbf{E}} = \begin{cases} \vec{0} & \text{above the planes} \\ \frac{\sigma}{\epsilon_o}\,\hat{\jmath} & \text{between the planes} \\ \vec{0} & \text{below the planes} \end{cases}$$

## 6 Electric Field Lines

**Ex. 56** Sketch the electric field lines for adisk of radius $R$ with a positive uniform surface charge density $\sigma$. Use what you know about the electric field very close to the disk and very far from the disk to make your sketch.

# 7 Electric Dipoles

## 7.1 Force and Torque on an Electric Dipole

$$\tau = (qE)\,(d\,\sin\phi)$$

$$\vec{p} = q\vec{d} \qquad \text{(electric dipole moment)}$$

$$\tau = pE\,\sin\phi \qquad\qquad \vec{\tau} = \vec{p}\times\vec{E}$$

Find the potential energy $U$ of an electric dipole $\vec{p}$ in an electric field $\vec{E}$. Since the electrostatic force is a *conservative* force, we can write:

$$W = -\Delta U \qquad\qquad dW = \tau\,d\theta = pE\,\sin\phi\,(-d\phi)$$

$$W = \int_{\phi_1}^{\phi_2} (-pE\,\sin\phi)\,d\phi = pE\,\cos\phi_2 - pE\,\cos\phi_2$$

Since $W = -\Delta U = U_1 - U_2$, we can write the potential energy function for an electric dipole in an electric field as:

$$U = -pE\,\cos\phi \qquad \text{or} \qquad U = -\vec{p}\cdot\vec{E}$$

This is the potential energy of an electric dipole $\vec{p}$ in an electric field $\vec{E}$.

**Ex. 59** Point charges $q_1 = -4.5$ nC and $q_2 = +4.5$ nC are separated by 3.1 mm, forming an electric dipole. a) Find the electric dipole moment (magnitude and direction). b) The charges are in a uniform electric field whose direction makes an angle of 36.9° with the line connecting the charges. What is the magnitude of this field if the torque exerted on the dipole has magnitude $7.2 \times 10^{-9}$ N·m?

**Problems from the homework**

**Prob. 69** Two positive point charges $Q$ are held fixed on the $x$-axis at $x = a$ and $x = -a$. A third positive point charge $q$, with mass $m$, is placed on the $x$-axis away from the origin at a coordinate $x$ such $|x| \ll a$. The charge $q$, which is free to move along the $x$ axis, is then released. a) Find the frequency of oscillation of the charge $q$. (*Hint:* Review the definition of simple harmonic motion in Section 13.2. Use the binomial expansion $(1+z)^n = 1 + nz + n(n-1)z^2/2 + \cdots$, valid for the case $|z| < 1$.) b) Suppose instead that the charge $q$ were placed on the $y$-axis at a coordinate $y$ such that $|y| \ll a$, and then released. If this charge is free to move anywhere in the $xy$-plane, what will happen to it? Explain you answer.

**Prob. 71** Two small spheres with mass $m = 15.0$ g are hung by silk threads of length $L = 1.20$ m from a common point (Fig. 21.36). When the spheres are given equal quantities of negative charge, so that $q_1 = q_2 = q$, each thread hangs at $\theta = 25.0^o$ from the vertical. a) Draw a digram showing the forces on each sphere. Treat the spheres as point charges. b) Find the magnitude of $q$. c) Both threads are now shortened to length $L = 0.600$ m, while the charges $q_1$ and $q_2$ remain unchanged. What new angle will each thread make with the vertical? (*Hint:* This part of the problem can be solved numerically by using trial values for $\theta$ and adjusting the values of $\theta$ until a self-consistent answer is found.)

**Prob. 86** A positive charge $Q$ is distributed uniformly along the $x$-axis from $x = 0$ to $x = a$. A positive point charge $q$ is located on the positive $x$-axis at $x = a + r$, a distance $r$ to the right of the end of $Q$ (Fig. 21.37). a) Calculate the $x$- and $y$-components of the electric field produced by the charge distribution $Q$ at points on the positive $x$-axis where $x > a$. b) Calculate the force (magnitude and direction) that the charge distribution $Q$ exerts on $q$. c) Show that if $r \gg a$, the magnitude of the force in part (b) is approximately $Qq/4\pi\epsilon_o r^2$. Explain why this results is obtained.

Answer:

$$E_x = \frac{k\lambda a}{x(x-a)} \quad \rightarrow \quad \frac{kQ}{x(x-a)}$$

**Prob. 94** Positive charge $Q$ is uniformly distributed around a semicircle of radius $a$ (Fig. 21.39). Find the electric field (magnitude and direction at the center of curvature $P$.

## Homework – Chapter 21

Exercises: 1, 4, 11, 19, 25, 27, 31, 36, 40, 41, 48, 50, 51, 55, 56, 59, 60, 66, 67
Problems: 69, 70, 71, 81, 86, 94, 98