Exam #2 PS250

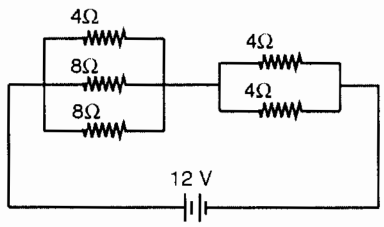
June 21, 2006 Dr. Darrel Smith

Show your work !!

Name: Answer Key

10 points

1. A resistor network is connected to a 12-V battery as shown in the figure to the right.



a. Calculate the equivalent resistance.

$\frac{1}{8} + \frac{1}{8} + \frac{1}{4} = \frac{1}{2} \Rightarrow 2\Omega$

$\frac{1}{4} + \frac{1}{4} = \frac{1}{2} \Rightarrow 2\Omega$

$2\Omega + 2\Omega = 4\Omega$

R_{eq} = 4 Ω

b. Calculate the current supplied by the battery.

$I = \frac{V}{R} = \frac{12\ V}{4\Omega} = 3A$

$I_{battery}$ = 3 A

c. Calculate the power supplied by the battery.

$P = VI = (12\ V)(3A) = 36$ Watts

$P_{battery}$ = 36 W

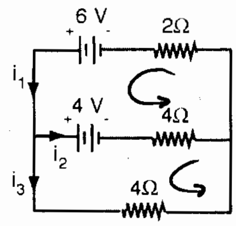
d. Calculate the current in one of the 8Ω resistors.

$I = \frac{V}{R} = \frac{6\ \text{volts}}{8\Omega} = 0.75\ A$

$I_{8\Omega}$ = 0.75 W

15 points

2. Applying Kirchhoff's rules to following circuit, calculate the following:
Hint: one of the currents is zero.



a. Calculate the currents i_1, i_2, and i_3.

$i_1 = i_2 + i_3$

Top loop: $6V - 4V - 4i_2 - 2i_1 = 0$

Bottom loop: $4V - 4i_3 + 4i_2 = 0$

Top: $2V = 2i_1 + 4i_2$

Bottom: $4V = -4i_2 + 4i_3$

Top: $2V = 6i_2 + 2i_3$

Bottom: $4V = -4i_2 + 4i_3$

Top: $-4V = -12i_2 - 4i_3$

Bottom: $4V = -4i_2 + 4i_3$

$0 = -16i_2$

$\boxed{i_2 = 0}$ $\boxed{i_1 = i_3}$

Top: $2V = 6 \times 0 + 2i_3$

$\boxed{i_3 = 1}$

i_1 = 1 A

i_2 = 0 A

i_3 = 1 A

b. Calculate the power delivered by the two batteries.

$P_{6V} = (6V)(1A) = 6\ W$

$P_{4V} = (4V)(0A) = 0\ W$

P_{6V} = 6 W

P_{4V} = 0 W

c. Show that the power dissipated by the resistors equals the total power supplied by the batteries.

$i_1^2(2\Omega) = 2\ W$

$i_2^2(4\Omega) = 0\ W$

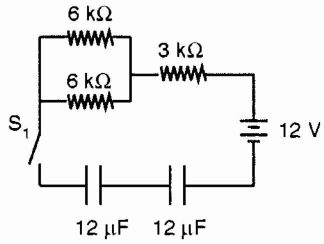
$i_3^2(4\Omega) = 4\ W$

Total Power Dissipated = 6 W

Same as the power supplied by the battery.

15 points

3. An *RC* circuit is shown in the figure to the right. The capacitors are initially uncharged. The switch S_1 is closed at $t = 0$ and the capacitors begin to charge.



a. Find the time constant for this circuit.

$$R_{eq} = \left(\frac{1}{6k\Omega} + \frac{1}{6k\Omega}\right)^{-1} + 3\,k\Omega = \underline{6\,k\Omega}$$

$$C_{eq} = \left(\frac{1}{12\mu F} + \frac{1}{12\mu F}\right)^{-1} = \underline{6\,\mu F}$$

$$\tau = RC = (6\,k\Omega)(6\,\mu F)$$

$$\tau = \underline{36\,ms}$$

$\tau =$ 36 ms

b. Find the *total* charge stored on the capacitors once they have become completely charged.

$$Q = VC_{eq} = 12V\,(6\mu F) = 72\,\mu C$$

$Q_{total} =$ 72 μC

c. Find the initial current immediately after S_1 is closed.

$$I_o = \frac{V}{R_{eq}} = \frac{12\,V}{6\times10^3\,\Omega} = \underline{2\,mA}$$

$I_o =$ 2 mA

d. Find the current at *72 ms*.

$$I = I_o e^{-t/RC} = 2\,mA\; e^{-72/36} = 2\,mA\,(e^{-2}) = \underline{0.271\,mA}$$

$I =$ 0.271 mA

e. What is the instantaneous power supplied by the battery at $t = 72\,ms$?

$$P = VI = (12\,V)(0.271\times10^{-3}\,A) = 3.25\times10^{-3}\text{ watts}$$

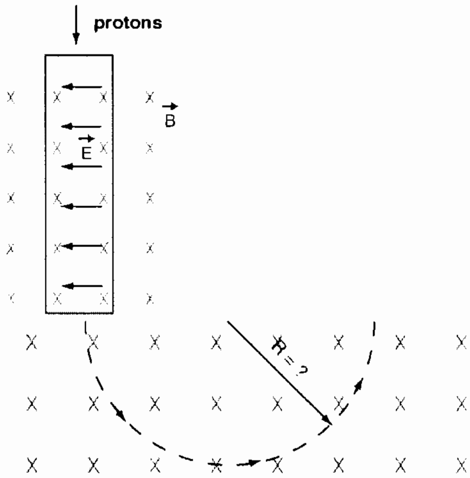
$$= \underline{3.25\,mW}$$

I from part d

$P =$ 3.25 mW

10 points

4. Protons enter a velocity selector shown in the figure to the right. The electric and magnetic fields in the velocity selector are 1.00×10^6 V/m and 1.00 T.



a. Calculate the velocity of the protons exiting the velocity selector.

$$\vec{F} = q\,(\vec{E} + \vec{v}\times\vec{B}) = \vec{0}$$ for protons traveling in a straight line.

$$v = \frac{E}{B} = \frac{1.00\times10^6\,V/m}{1.00\,T} = 10^6\,m/s$$

$v =$ 10^6 m/s

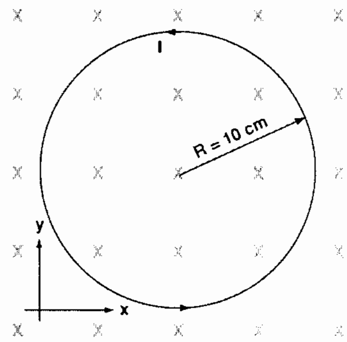
b. After leaving the velocity selector, the protons enter region containing a constant, homogeneous magnetic field of 0.1042 T. Find the radius of curvature of the protons as they begin to orbit in the magnetic field.

$$R = \frac{mv}{qB} = \frac{(1.67\times10^{-27}\,kg)(10^6\,m/s)}{(1.602\times10^{-19}\,C)(0.1042\,T)} = 0.100\,m$$

$R =$ 10 cm

15 points

5. A current loop 10 cm in radius carries a current of 2.00 A as shown in the figure to the right.



a. Calculate its magnetic dipole moment.

$$\mu = IA = (2.00\ A)\,\pi(0.100\ m)^2 = 6.28 \times 10^{-2}\ A\cdot m^2$$

$\mu =$ 6.28×10^{-2} $A\cdot m^2$

b. The current loop is inserted in a homogenous magnetic field of 1.30 T and is initially in its "highest" potential energy orientation. How much work is done by the magnetic field to "flip" the magnetic dipole moment 180° to its "lowest" potential energy orientation?

$$W = -\Delta U = -(U_f - U_i) = -(-\mu \cdot B\cos\theta_f - (-\mu B\cos\theta_i))$$

$$W = \mu B(\cos\theta_f - \cos\theta_i) = \mu B(\cos 0° - \cos 180°) = 2\mu B$$

$$W = 2(6.28\times10^{-2}\ A\cdot m^2)(1.30\ T) = 0.163\ J$$

$W_{mag.\ field} =$ 0.163 J

c. What is the maximum torque applied to the magnetic dipole while it is "flipping" from its initial position to its final position? This occurs when $\theta = 90°$

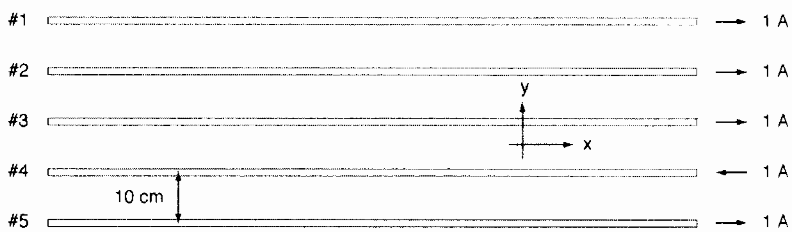
$$\vec{\tau} = \vec{\mu}\times\vec{B} \qquad \tau = \mu B\sin\theta = \mu B\sin 90° = \mu B = (6.28\times10^{-2})(1.30)$$

Note: There's an ambiguity in the direction of the torque. It could be in either $+\hat{j}$ or $-\hat{j}$ direction.

$\vec{\tau} = ($ 0 $\hat{i} +$ 8.17×10^{-2} $\hat{j} +$ 0 $\hat{k})\ N\cdot m$

10 points

6. Currents are flowing through infinitely-long, equally-spaced, parallel wires as wires shown in the figure to the right.



$$F = ILB = IL\frac{\mu_0 I}{2\pi R} \qquad \frac{F}{L} = \frac{\mu_0 I^2}{2\pi R}$$

a. Calculate the *force per unit length* on wire #2.

(#1 and #3 cancel)

$$\left(\frac{F}{L}\right)_2 = +\frac{\mu_0 I^2}{2\pi(.20)} - \frac{\mu_0 I^2}{2\pi(.30)} = \frac{\mu_0 I^2}{2\pi}\left(\frac{1}{0.2} - \frac{1}{0.3}\right)$$

$$2\times10^{-7}(1)^2(1.67) = 3.33\times10^{-7}$$ (upward)

$F/L =$ $+3.33\times10^{-7}$ $\hat{j}$ N/m

b. Calculate the *force per unit length* on wire #4.

$$\left(\frac{F}{L}\right)_4 = \frac{\mu_0 I^2}{2\pi(0.20)} + \frac{\mu_0 I^2}{2\pi(0.30)} = \frac{\mu_0 I^2}{2\pi}\left(\frac{1}{0.2} + \frac{1}{0.3}\right)$$

(#3 and #5 cancel)

$$2\times10^{-7}(1)^2(8.33) = 1.67\times10^{-6}$$ (downward)

$F/L =$ -1.67×10^{-6} $\hat{j}$ N/m

c. Calculate the *force per unit length* on wire #3.

$$\left(\frac{F}{L}\right)_3 = \frac{\mu_0 I^2}{2\pi(0.10)} + \frac{\mu_0 I^2}{2\pi(0.10)} = \frac{\mu_0 I^2}{2\pi}\left(\frac{2}{0.10}\right) = \frac{2\times10^{-7}(1)^2 2}{0.10} = 4\times10^{-6}$$ (upward)

$F/L =$ 4×10^{-6} $\hat{j}$ N/m

(#1 and #5 cancel)