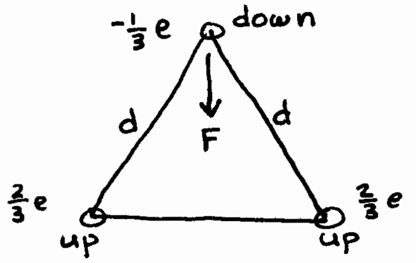
PS250 Exam #1
June 6, 2005 Dr. Darrel Smith

Please, show your work !!

Answer Key
Name

10 Points

1. Three quarks, two *up* quarks and one *down* quark form the constituent charged particles of a proton. If the three quarks are positioned on the vertices of an equilateral triangle, calculate the net force acting on the one *down* quark. The charge on each of the *up* quarks is +2/3 *e* while the charge on the *down* quark is -1/3*e*. Assume the side of the triangle are 1.00×10^{-15} m.



$$F = \frac{k\left(\frac{2}{3}e\right)\left(\frac{1}{3}e\right)}{d^2} \times (2) \times \cos 30°$$

$$F = \frac{8.988\times10^{9}\left(\frac{4}{9}\right)\left(1.602\times10^{-19}\right)\cos 30°}{\left(1.00\times10^{-15}\right)^2}$$

$F = 88.8$ N

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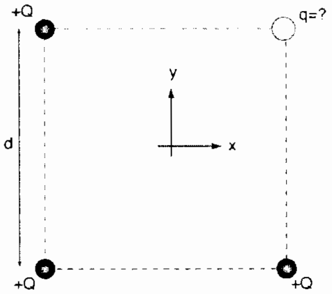
5 points

2. Four charges are held in place on the corners of a square as shown in the figure to the right.



a.) Find the value of *q* if the total potential energy of the 4-charge system is zero. Write your answer in terms of *Q*.

$$U = 2\frac{kQ^2}{d} + \frac{kQ^2}{\sqrt{2}d} + \frac{2kqQ}{d} + \frac{kQq}{\sqrt{2}d} = 0$$

$$2Q^2 + \frac{Q^2}{\sqrt{2}} + 2qQ + \frac{qQ}{\sqrt{2}} = 0 \qquad Q\left(2+\frac{1}{\sqrt{2}}\right) + q\left(2+\frac{1}{\sqrt{2}}\right) = 0 \qquad q = -Q$$

$q =$ -1 Q

5 points (extra credit)

b.) What is the magnitude of the electrostatic force on *q*? Write your answer in terms of kQ^2/d^2.

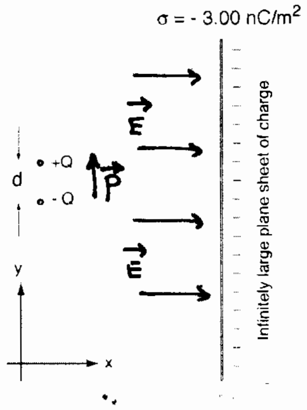
$$\hat{i}\left(\frac{kqQ}{d^2} + \frac{kqQ}{2d^2}\frac{1}{\sqrt{2}}\right) + \hat{j}\left(\frac{kqQ}{d^2} + \frac{kqQ}{2d^2}\frac{1}{\sqrt{2}}\right) = \hat{i}\,\frac{kqQ}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right) + \hat{j}\,\frac{kqQ}{d^2}\left(1+\frac{1}{2\sqrt{2}}\right)$$

$$\vec{F} = -\frac{kQ^2}{d^2}\left(1.354\,\hat{i} + 1.354\,\hat{j}\right) \Rightarrow |\vec{F}| = \frac{kQ^2}{d^2}\sqrt{2}\,(1.354)$$

$F =$ 1.914 kQ^2/d^2

15 points

3. A dipole is held in a position parallel to an infinitely large plane sheet with a surface charge density of -3.00 nC/m². The dipole is made of two equal but oppositely charged particles of 3.00 nC separated by a distance of 5.00 cm. If the dipole is located sufficiently far from the infinite sheet of charge (i.e., the dipole does not disturb the charge distribution on the *infinite* sheet), find the following:



a. The electric field at the location of the dipole,

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{3.00\ nC/m^2}{2\left(8.85\times10^{-12}\ C^2/N\cdot m^2\right)} = 169.5\ N/C$$

$\vec{E} =$ 169.5 $\hat{i}$ N/C

b. The torque acting on the dipole,

$$\vec{\tau} = \vec{p} \times \vec{E} \Rightarrow \tau = pE \underbrace{\sin\theta}_{=1}$$

$$\tau = pE = (3nC)(5\times10^{-2} m)(169.5 N/C) = 2.54\times10^{-8} N\cdot m$$

$\vec{\tau}$ = -2.54×10^{-8} $\hat{k}$ N·m

c. The change in potential energy when the dipole is release and becomes aligned with the electric field.

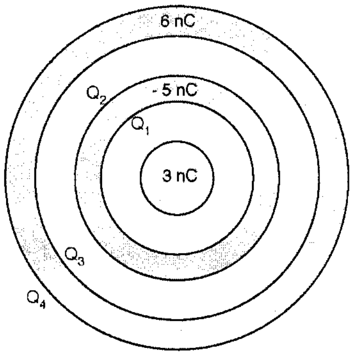
$$\Delta U = U_f - U_i = -\vec{p}_f\cdot\vec{E} - (-\vec{p}_i\cdot\vec{E}) = -pE$$

$$\Delta U = -(3nC)(5\times10^{-2} m)(169.5 N/C) = -2.54\times10^{-8} N\cdot m$$

ΔU = -2.54×10^{-8} J

15 points

4. Two concentric conducting spheres surround a solid conducting sphere as shown in the figure.



a. Find the charges Q_1, Q_2, Q_3, and Q_4 on the *inner* and *outer* surfaces of the spherical shells.

Q_1 = -3 nC

Q_2 = -2 nC

Q_3 = +2 nC

Q_4 = +4 nC

b. Find the magnitude of the electric field on the outer surface of the largest sphere if its outer diameter is 40.0 cm.

$$Q = +4 nC \qquad E = \frac{kQ}{R^2} = \frac{8.988\times10^9 (4\times10^{-9} C)}{(0.20 m)^2} = 899 V/m$$

E = 899 V/m

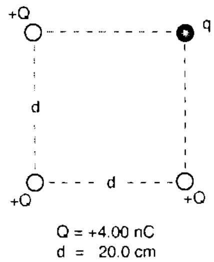
c. What is the potential inside the outermost concentric sphere if the potential V is zero at infinity?

$$V_{\text{on the surface}} = RE = 0.20m\,(899 V/m) = 180 V$$

V = 180 volts

15 points

5. Four charges are held in place at the corners of a square as shown in the figure to the right.



a. Calculate the potential at the *"location of q"* due to the other three charges.

$$V = 2\frac{kQ}{d} + \frac{kQ}{\sqrt{2}\,d} = \frac{kQ}{d}\left(2 + \frac{1}{\sqrt{2}}\right) = \frac{8.988\times10^9 (4.0\times10^{-9})}{0.20}(2.71)$$

$$V = 487 \text{ Volts}$$

V = 487 volts

b. Calculate the potential energy of q if q = +2.00 nC.

$$U = qV = (2.00\times10^{-9} C)(487 \text{ volts}) = 9.73\times10^{-7} J$$

PE = 9.73×10^{-7} J

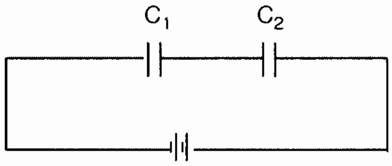
c. If q is released from rest, what will the magnitude of its velocity be once it is far away from the other three charges? Assume the mass of q is 5.00×10^{-5} kg.

$U = \frac{1}{2}mv^2 \qquad v = \sqrt{\frac{2U}{m}} = \sqrt{\frac{2(9.73\times10^{-7}\text{ J})}{5\times10^{-5}\text{ kg}}} \qquad v = 0.197\text{ m/s}$

$v =$ 0.197 m/s

25 points

6. Two identical capacitors are connected in series and connected to a 12-battery as shown in the figure. $C_1 = C_2 = 8.00\ \mu F$



a. Find the total potential energy stored in the two capacitors once they are fully charged.

$C_{eq} = \left(\frac{1}{8\mu F} + \frac{1}{8\mu F}\right)^{-1} = \left(\frac{1}{4\mu F}\right)^{-1} = 4\mu F$

$PE = U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}(4\times10^{-6}F)(12\text{ V})^2 = 2.88\times10^{-4}\text{ J}$

$PE_{total} =$ 2.88×10^{-4} J

b. How much charge is on each plate of the capacitors?

$Q = VC_{eq} = 12\text{ V}(4\mu F) = 48\mu C$

$Q_{each\ plate} =$ 48 μC

c. A dielectric with dielectric constant $K=3.00$ is inserted in the second capacitor. Find the total potential energy stored in the two capacitors once they are fully charged.

$C_{eq} = \left(\frac{1}{8\mu F} + \frac{1}{24\mu F}\right)^{-1} = \left(\frac{4}{24\mu F}\right)^{-1} = 6\mu F$

$PE = U = \frac{1}{2}CV^2 = \frac{1}{2}(6\mu F)(12\text{ V})^2 = 4.32\times10^{-4}\text{ J}$

$PE_{total} =$ 4.32×10^{-4} J

50% more energy than part (a).

d. How much charge is on the plate of the first capacitor?

$Q = VC_{eq} = (12\text{ V})(6\mu F) = 72\mu C$

$Q_{C1} =$ 72 μC

e. What is the voltage drop across the first capacitor?

$V_{drop} = \frac{Q}{C_1} = \frac{72\mu C}{8\mu F} = 9\text{ volts}$

$V_{C1} =$ 9 volts

Useful constants: $k = 8.988 \times 10^9\ N\cdot m^2/C^2 \qquad \varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N\cdot m^2$
$e = 1.602 \times 10^{-19}$ C