Chapter 27
Magnetic Fields and Magnetic Forces

In this chapter we investigate forces exerted by magnetic fields. In the next chapter we will study the sources of magnetic fields. The force produced by magnetic fields has played a pivotal role in our production of electrical energy. Likewise, the transport of this electrical energy (to produce magnetic fields far away from their source) has made it more convenient to live and work at remote locations. We will also introduce the concept of magnetic flux ($\Phi_{\text{mag}}$) and the important role it plays in the production of electrical power.

1 Magnetism

The magnetic phenomena was first recorded at least 2500 years ago in fragments of magnetised iron ore found near the ancient city Magnesia (Manisa, in western Turkey). Today we call these fragments permanent magnets.

Magnetic fragments were observed to attract and repel when brought close together.

![Figure 1: This figure shows the possible forces of attraction and repulsion that occur with bar magnets.](image)

Before the source of magnetic fields was understood, the interaction of permanent magnets and compass needles were described in terms of magnetic poles.
The Earth’s Magnetic Field

Figure 2: This figure shows the earth’s magnetic field with the *internal* south-pole in the northern hemisphere. The north-seeking compass needles point northward to the earth’s magnetic south-pole.

1.1 Magnetic Poles Versus Electric Charge

In contrast to electric charges, magnetic poles always come in pairs and can’t be isolated.

![Breaking a magnet in two](image)

... yields two magnets, not two isolated poles.

Figure 3: This figure shows that trying to separate the *north* pole from the *south* pole is a futile exercise. It always results in the production of two magnetic dipoles.

To date: **magnetic monopoles** have not been observed!!

The Danish physicist, Hans Oersted, first noticed the connection between current and the production of magnetic fields in 1820.
Figure 4: This is the apartment in Copenhagen, Denmark where Hans Christian Oersted made his discovery that there was a connection between currents and magnetic field.

Figure 5: This figure shows the effect of a nearby current and its interaction with a compass needle. Opposite currents result in an opposite change in the direction of the compass needle.
2 Magnetic Field

We would like to introduce the concept of a magnetic field much in the same way that we described the electrostatic field.

1. We said that the distribution of electric charge will create an electric field $\vec{E}$ in the surrounding space.
2. Also, we observed that the electric field exerted a force proportion to the product of the charge and the electric field $\vec{F} = q\vec{E}$.

In the case of magnetic interactions, we should be able to say the following:

1. A moving charge or a current creates a magnetic field in the surrounding space, and
2. The magnetic field exerts a force $\vec{F}$ on other moving charges or currents that are present in the field.

2.1 Magnetic Forces on Moving Charges

$$\vec{F} = q \vec{v} \times \vec{B}$$ (1)

where $q$ is the charge (coulombs), $v$ is the velocity (m/s), and $B$ is the magnetic field (teslas). One tesla is:

$$1 T = 1 N/(A \cdot m)$$

Figure 6: This figure illustrates the magnetic force resulting from a charge $q$ moving with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$. 

(b)

A charge moving at an angle $\phi$ to a magnetic field experiences a magnetic force with magnitude $F = |q|v_1B = |q|vB \sin \phi$. $\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. 


2.2 The Right-Hand Rule

Figure 7: This figure illustrates how to apply the right-hand rule to the equation $\vec{F} = q\vec{v} \times \vec{B}$.

Figure 8: This figure illustrates how opposite charges result in opposite force vectors $\vec{F}$. This figure illustrates how opposite charges result in opposite force vectors $\vec{F}$. 
**Ex. 27.6:** An electron moves at $1.40 \times 10^6$ m/s through a region in which there is a magnetic field of unspecified direction and magnitude $7.50 \times 10^{-2}$ T. (a) What are the largest and smallest possible magnitudes of the acceleration of the electron due to the magnetic field? (b) If the actual acceleration of the electron is one-fourth of the largest magnitude in part (a), what is the angle between the electron velocity and the magnetic field?

### 2.3 Measuring Magnetic Fields with Test Charges

![Diagram](image.png)

Figure 9: This figure illustrates how to measure the strength and direction of a magnetic field by tracking the motion of a test charge $q$.

$$\vec{F} = q \vec{v} \times \vec{B} = m\vec{a}$$
3 Magnetic Field Lines and Magnetic Flux

Figure 10: The magnetic fields are stronger at the pole faces (i.e., higher flux) and weaker at field points far away from the pole face (i.e., lower flux).
3.1 Magnetic Flux and Gauss’s Law for Magnetism

Figure 11: This figure shows the continuity of magnetic field lines for a number of different geometries. Notice that there are no sources and no sinks for the magnetic filed lines.

\[
\Phi_B = \int B \cos \phi \, dA = \int B_{\perp} \, dA = \int \vec{B} \cdot d\vec{A} \tag{2}
\]

Figure 12: This figure illustrates how to calculate the flux of magnetic field \( \Phi_m \) passing through an open surface.

If we have a planar surface, then the flux is easily written as:

\[
\Phi_B = B_{\perp} A = B A \cos \phi \tag{3}
\]
The unit of flux is called the **weber** (1 Wb), in honor of the German physicist Wilhelm Weber (1804-1891).

\[
1 \text{ Wb} = 1 \text{T} \cdot \text{m}^2 = 1 \text{N} \cdot \text{m} / \text{A}
\]

**Gauss’s Law for Magnetism**

The total magnetic flux through any **closed surface**:

\[
\Phi_B = \oint_S \vec{B} \cdot d\vec{A} = 0 \quad \text{(Always)}
\]

**Magnetic Flux Through a Surface**

![Figure 13](image)

Figure 13: This figure illustrates how to calculate the flux of magnetic field \( \Phi_m \) passing through an **open** surface. In this case, the magnetic flux \( \Phi_B \neq 0 \).

### 4 Motion of Charged Particles in a Magnetic Field

\[
\vec{F} = q\vec{v} \times \vec{B} \quad \text{(4)}
\]

Motion of a charged particle under the action of a magnetic field alone is always motion with constant speed.

The angular velocity of the charged-particle’s circular motion can be found by using Newton’s 2nd law:
(a) The orbit of a charged particle in a uniform magnetic field

A charge moving at right angles to a uniform $\vec{B}$ field moves in a circle at constant speed because $\vec{F}$ and $\vec{v}$ are always perpendicular to each other.

Figure 14: This figure illustrates the forces acting on a positive charge as it moves with uniform circular motion perpendicular to magnetic field.
\[ \sum F_{ext} = ma_c \quad \rightarrow \quad qvB = \frac{mv^2}{R} \]

\[ p = mv = qBR \quad \text{Also} \quad v = R\omega \]

So,

\[ \omega = \frac{v}{R} = \frac{|q|B}{m} \quad \text{(cyclotron frequency)} \quad (5) \]

**Magnetic Bottle in Plasma Physics**

Figure 15: This figure demonstrates how a magnetic field can be used to contain high-temperature ions in a *magnetic bottle*. 
Figure 16: This figure shows a gamma-ray conversion into an electron-positron pair, along with a $\delta$-ray (electron) kicked out of a neutral hydrogen atom during the conversion process. At first glance, it appears that charge is not conserved; however, if you include the $\delta$-ray electron in the initial part of the “balanced charge” equation, charge is conserved. $(\gamma + e^- \rightarrow e^+ e^- + e^-)$
Velocity Selector

If electric and magnetic fields are simultaneously applied to the motion of a charged particle (as shown in the figure below), it is possible to build a velocity selector:

\[ \vec{F} = q \left( \vec{E} + \vec{v} \times \vec{B} \right) \]

(a) Schematic diagram of velocity selector

Figure 17: This figure illustrates how to calculate the flux of magnetic field \( \Phi_m \) passing through an open surface.

The ions that successfully make it through the collimator at the bottom of the velocity selector satisfy the following condition (ignore gravity):

\[ v = \frac{E}{B} \quad \text{(ions that move in a straight line)} \]  

(6)
Mass Spectrometers

A mass spectrometer is shown in the following figure. This device is used to separate nuclear isotopes with slightly different atomic masses (e.g., $^{235}\text{U}$ and $^{238}\text{U}$.)

![Mass Spectrometer Diagram](image)

**Figure 18:** This figure shows ions in a Bainbridge mass spectrometer moving with the same velocity but different masses being separated in the second magnetic field at the bottom of the figure. In this figure, the magnetic field $\vec{B}$ is presumed to be the same inside and outside the velocity selector.

\[ v = \frac{E}{B} \quad R = \frac{mv}{|q|B} \]

\[ R = \frac{mE}{|q|B^2} \]

**(Radius as a function of mass)** (7)
27.34: In the Bainbridge mass spectrometer (see Fig. 27.24 in the text), the magnetic-field magnitude in the velocity selector is 0.510 T, and ions having a speed of $1.82 \times 10^6$ m/s pass through undeflected. (a) What is the electric-field magnitude in the velocity selector? (b) If the separation of the plates is 5.20 mm, what is the potential difference between the plates?

5 Magnetic Force on a Current-Carrying Conductor

We can compute the force on a current-carrying conductor starting with the magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ on a single moving charge. Let’s assume that all the positive charges are moving with a uniform velocity $v_d$.

![Diagram](image)

Figure 19: This figure shows the current moving “upwards” in a uniform magnetic field $\vec{B}$ pointing into the page. The resulting force $\vec{F} = qv_d \times \vec{B}$ points to the left.

$$F = (nA\ell)(qv_dB)$$
where \( n \) is the number density of positive charges, \( A \) is the cross-sectional area, and \( \ell \) is the “short” segment length of wire. If we identify the current density as \( J = nqv_d \) and \( JA = I \), we can rewrite the above equation as:

\[
F = I\ell B
\]

If the magnetic field \( \vec{B} \) is not perpendicular to the wire but makes an angle \( \phi \) with it, then \( F = I\ell B \sin \phi \). This equation can be written using vector notation as:

\[
\vec{F} = I\vec{\ell} \times \vec{B} \tag{8}
\]

where \( \vec{\ell} \) points in the current direction. If the conductor is not straight, we can divide it into infinitesimal segments \( d\vec{\ell} \). The differential force due to this differential line segment will be:

\[
d\vec{F} = I\,d\vec{\ell} \times \vec{B} \tag{9}
\]

where, once again, \( d\vec{\ell} \) points in the direction of the current.

![Figure 20](image)

**Figure 20**: This figure shows the force on a current-carrying wire of length \( \ell \) subtending an angle \( \phi \) with respect to the magnetic field \( \vec{B} \).

- Magnitude is \( F = llB_\perp = llB \sin \phi \).
- Direction of \( \vec{F} \) is given by the right-hand rule.
Ex. 27.39: A long wire carrying 4.50 A of current makes two 90° bends as shown in the figure. The bent part of the wire passes through a uniform 0.240 T magnetic field directed as shown in the figure and confined to a limited region of space. (a) Find the magnitude of the force that the magnetic field exerts on the wire. (b) Find the direction of the force that the magnetic field exerts on the wire.

Figure 21: This figure is used to determine the force on a current-carrying wire making two 90° bends in a uniform magnetic field.
5.1 Magnetic Force on a Curved Conductor

Figure 22: This figure is used to discuss the total magnetic force on this current loop and segment.

\[ dF_x = IR \, d\theta \, B \cos \theta \quad \quad dF_y = IR \, d\theta \, B \sin \theta \]

\[ F_x = \int dF_x = IRB \int_0^\pi \cos \theta \, d\theta = 0 \]
\[ F_y = \int dF_y = IRB \int_0^\pi \sin \theta \, d\theta = 2IRB \]
6 Force and Torque on a Current Loop

Figure 23: This figure shows how to calculate the forces on each of the four segments of the current loop. From this, the resulting torques can be calculated.

The force on each segment is going to be:

\[ \vec{F} = I \vec{\ell} \times \vec{B} \]

While the sum of the forces is zero, the sum of the torques is generally not.

\[ \sum \vec{F}_i = 0 \quad \text{however, in general} \quad \sum \vec{\tau} \neq \vec{0} \]

\[ \tau = 2F(b/2) \sin \phi = (IBa)(b \sin \phi) \]

The magnitude of the magnetic torque on a current loop is:

\[ \tau = IBA \sin \phi \quad (10) \]

where \( A \) is the area of the loop and \( \phi \) is the angle between the area-normal \( \vec{A} = A\hat{n} \) and the magnetic field \( \vec{B} \). The product \( IA \) is called the magnetic dipole moment \( \vec{\mu} \). So, we can write the torque as
Torque on a Magnetic Dipole

\[ \vec{\tau} = \vec{\mu} \times \vec{B} \]  \hspace{1cm} \text{(Torque on a magnetic dipole)} \hspace{1cm} (11)

where \( \vec{\mu} = I\vec{A} \)

Potential Energy for a Magnetic Dipole

Similar to the potential energy for an electric dipole in an electric field where \( U = -\vec{p} \cdot \vec{E} \), we have the potential energy for a magnetic dipole in a magnetic field is:

\[ U = -\vec{\mu} \cdot \vec{B} \] \hspace{1cm} (12)

27.49: A coil with a magnetic moment of 1.50 A·m\(^2\) is oriented initially with its magnetic moment antiparallel to a uniform magnetic field of magnitude 0.840 T. What is the change in potential energy of the coil when it is rotated 180° so that its magnetic moment is parallel to the field?

7 Direct Current Motor

Electric motors play an important role in small “stepping” motors to large industrial motors. In a DC electric motor a magnetic torque acts on a current-carrying conductor, and electrical energy is converted to mechanical energy.

The moving part of the motor is called the rotor. The ends of the rotor wires are attached to circular conducting segments that form a commutator. Each of the two commutator segments makes contact with one of the terminals, or brushes which, in turn, are connected to an emf source.
Figure 24: This figure shows the principle design of a dc motor (direct current). As the loop is rotating, the torque varies but always in a direction to cause the loop to continuously turn in the counter-clockwise direction.

8 The Hall Effect

The current density $J_x$ is in the $x$-direction, and the magnetic force ($\vec{F} = q\vec{v} \times \vec{B}$) acts on the moving electrons causing them to move toward the “top” of the strip. This motion continues until the electric field set up by the separation of charge results in an equal-and-opposite force $\vec{F} = q\vec{E}$ towards the bottom of the strip. The resulting potential difference between the top and the bottom of the strip can be used to measure magnetic fields (e.g., Hall probes).

Looking at Fig. 25, we can write the following for the voltage between the “top” and the “bottom” of the plate where $w$ is the width of the conducting plate in the $z$ direction:

$$V = E_e w \quad \Rightarrow \quad V = v B w$$

Using the relationships $J = I_x/A = nev$, and $A = tw$, we can write:

$$V_z = \frac{I_x B_y}{nte}$$
where \( n \) is the electron charge density, \( t \) is the thickness in the \( y \)-direction, and \( e \) is the electric charge.

(a) Negative charge carriers (electrons)

The charge carriers are pushed toward the top of the strip ...

... so point \( a \) is at a higher potential than point \( b \).

Figure 25: This figure shows the forces on the charge carriers (e.g., electrons) as a current moves through conducting plate perpendicular to a magnetic field. Electrons drift to the top of the plate producing a potential difference between the top and the bottom of the plate. The separation between the positive and negative charges is \( w \), or the height of the plate as shown in the figure.