Millikan Oil Drop Experiment

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The purpose of this experiment is to use Millikan's oil drop apparatus to observe the quantization of the electron *charge*, and to measure the fundamental charge of the electron $e = 1.602 \times 10^{-19}\ C$. Two methods are described in the Instruction Sheet on my website; however, only the first method will be used, and this method is described in the text below. The method involves two measurements of each oil drop–the free-fall terminal velocity, and the voltage required to suspend the charged droplet motionless between the capacitor plates. The combination of these two measurements will be used by students to calculate the amount of charge (i.e., *coulombs*) on each of the oil droplets. Students are expected to find evidence for charge quantization, and also determine the value of the elementary charge (e).

I. BACKGROUND

Robert Andrews Millikan constructed an experiment where oil was atomized (sprayed through a fine nozzle) to form tiny droplets between two oppositely charged parallel metal plates. Some of the oil droplets pick up one or more excess electrons while passing through the atomizer. With the application of an electric field E, the motion of the charged droplets can either (1) remain motionless (due to an applied E-field), or (2) execute free-fall (with no E-field aplied).

When Robert Millikan made this historic measurement (1909), scientists had already investigated the charge-to-mass ratio (e/m) of the electron. While the e/m ratio was important for steering electron beams ($a = eE/m$), it did not reveal either the "mass" or the "charge" of the electron. The observation of charge quantization and its fundamental value (e) led scientists to a new discovery, namely, electric charge was discrete (i.e., $0e$, $\pm e$, $\pm 2e, \ldots$) and this discrete nature is found in all "charged" elementary particles (e.g., protons, electrons, π^+, π^-, $\ldots$). Moving forward into the future (1960-1970), it was discovered that quarks and antiquarks ($q, \bar{q}$) had fractional units of charge ($\pm\frac{1}{3}e$, and $\pm\frac{2}{3}e$).

II. THEORY

In this application of the Millikan oil drop experiment, students will apply a combination of forces to each oil droplet and observe their motion. **Take note**–the charge Q on each droplet can be different because more than one electron ($q = -e$) may be stripped off the oil droplet during the atomizing process. The student is expected to measure the charge (Q) of ≈30 oil droplets to acquire sufficient statistics. The radius of each droplet should be recorded because a final correction will be applied to each measured charge ($Q \rightarrow Q_c$) before plotting the data on a histogram. Students should make the following measurements for each oil droplet: (1) a static measurement, and (2) a terminal velocity measurement. The *static* and *terminal velocity* measurements will be used to determine the radius r and charge Q for each oil droplet.

A. The static measurement

The charged oil droplet is held motionless while under the influence of an electric field ($E = U/d$) where U is the voltage applied to the capacitor plates, and d is the separation between the plates. Since the drop is motionless, it obeys Newton's 1st law ($\sum \vec{F}_i = \vec{0}$) resulting in the following equation:

$$QE - mg = 0\ . \tag{1}$$

With the oil droplet suspended motionless, it is affected by three forces: (1) the gravitational force $m_{oil}g = \rho_{\text{oil}}\, g\, V_{\text{droplet}}$, (2) the buoyant force $\rho_{\text{air}} g V_{\text{droplet}}$, and (3) the electric force QE. Combining (1) and (2), the apparent weight of the droplet (mg) can be written as $\rho g V_{\text{droplet}}$, where ρ is the "net" density $\rho = \rho_{oil} - \rho_{\text{air}}$.

$$mg = \rho g V_{\text{droplet}} \tag{2}$$

Using Eq. 1, there is a relationship between the charge Q and the mass m:

$$Q = \frac{g}{E} m\ . \tag{3}$$

So, we need to determine the mass m of each oil droplet in order to determine the charge Q on each oil droplet.

B. The oil droplet in free-fall

To determine the mass m of each oil droplet, we will use the *free-fall* technique where we measure the *terminal velocity* of each oil droplet. When the oil droplet is in free-fall, it reaches terminal velocity v_1 rather quickly due to the air-resistance. The force due to the air resistance is called the Stoke's force ($F_{\text{AR}} = 6\pi\eta r v_1$), where η is the viscosity of the air as the oil droplet descends at terminal velocity, r_1 is the radius of the droplet, and v_1

is the *terminal velocity*. Once again, the motion of the oil droplet is described by Newton's 1st law:

$$6\pi\eta r v_1 \; - \; mg \; = \; 0 \; . \tag{4}$$

Take note that as the droplets *fall* under the influence of gravity, they appear as "points of light" to be *rising* when observed through the eyepiece.

C. Method 1

The two techniques used in Method 1 are described above. We will not be using Method 2. The *terminal velocity* technique is used to determine the radius r and thus, the mass m of each oil droplet, and the associated equations are shown below:

$$mg - 6\pi r v_1 \eta = 0$$

$$V\rho g - 6\pi r v_1 \eta = 0$$

$$\frac{4}{3}\pi r^3 \rho g \; - \; 6\pi r v_1 \eta = 0$$

$$r = \sqrt{\frac{9\eta v_1}{2\rho g}} \tag{5}$$

Using this as the radius of the droplet, the charge can be calculated by the following relation:

$$Q = \frac{6\pi d\eta v_1}{U_1}\sqrt{\frac{9\eta v_1}{2\rho g}} \tag{6}$$

Where

$$\eta = 1.81 \times 10^{-5} \; \left[\frac{Ns}{m^2}\right]$$

$$d = 6 \times 10^{-3} \; m$$

$$\rho_{oil} = 875.3 \; \frac{kg}{m^3}$$

$$\rho_{\text{air}} = 1.29 \; \frac{kg}{m^3}$$

$$\rho = 874 \; \frac{kg}{m^3}$$

$$g = 9.795 \; \frac{m}{s^2}$$

This yields a final equation of

$$Q = 2 \times 10^{-10} \, \frac{v_1^{\frac{3}{2}}}{U} \;\; \text{coulombs} \tag{7}$$

D. Method 2–We will not use this technique

The second method requires both timing the free fall of the droplet and measuring a rise time for the droplet under the influence of an electric field.

Using this technique, the charge Q can be calculated using the following equation:

$$Q = (v_1 + v_2)\frac{\sqrt{v_1}}{U_2}\eta^{3/2}\frac{18\pi a}{2\rho g} \tag{8}$$

where v_2 is the *rising* velocity of the oil droplet. This equation can be simplified using the same constants as method 1:

$$Q = (v_1 + v_2)\frac{v_1}{U_2}(2 \times 10^{-10} C) \tag{9}$$

E. Correction Factor

No matter which method you use, the temperature and air pressure affect different sized oil droplets. As a result, a correction factor must be introduced to include the temperature and pressure. The corrected charge Q_{c} is given by:

$$Q_c = \frac{Q}{(1 + \frac{b}{rP})^{3/2}} \tag{10}$$

where P is the pressure, r is the radius of the droplet, and b is a constant. The reason for including this correction is due to the size of the oil droplet. The size of the droplets are so small that the motion of the air molecules begin to influence their motion when undergoing *free-fall*. You can imagine that if the oil droplet becomes very small, it will become suspended in the air (e.g., like small drops of water in foggy weather).

III. OBJECTIVE

Once again, the purpose of this lab is to observe the *quantization of charge* and determine the *size* of the fundamental charge, e.

IV. EQUIPMENT

A Millikan oil-drop apparatus is shown in Fig. 1. A fine spray of oil is injected in the region between the horizontal capacitor plates that are connected to an external power supply. The motion of the oil drops are observed using an eyepiece with graduated cross-hairs. The operation of the oil-drop apparatus is described in the operations manual for this experiment.

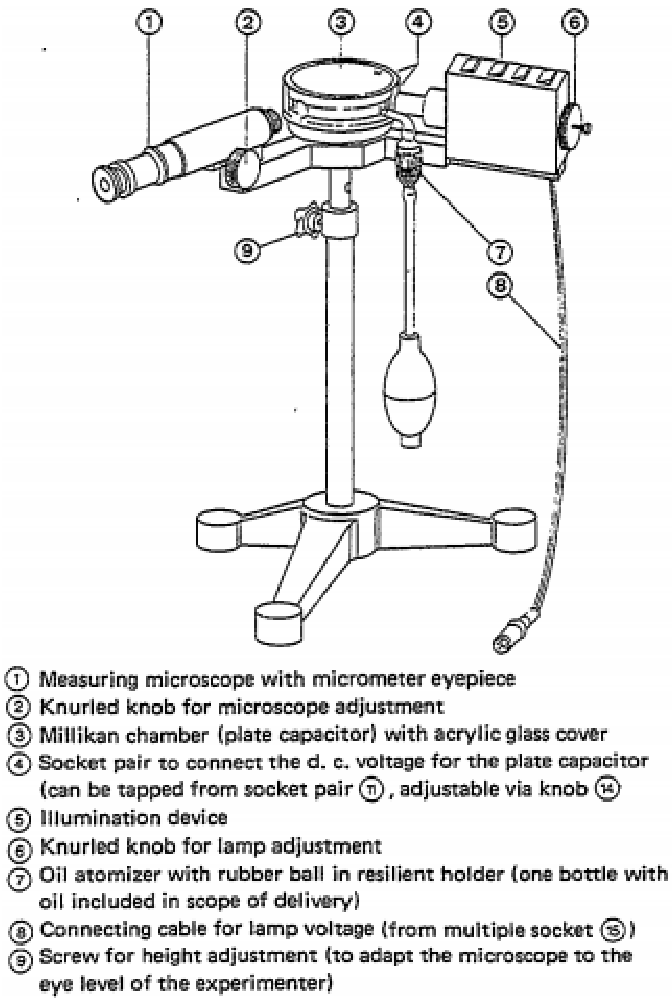

FIG. 1. Millikan Apparatus

V. DATA ACQUISITION

The physical measurements collected in this experiment include the voltage U, the free-fall distance S_1, and the free-fall time t_1. Make sure to include the uncertainties of these measurements in your log book. Make multiple measurements for each droplet and gather data for ~ 30 droplets. Leave columns in your table to calculate the radius, the charge Q, and the corrected charge Q_C for each droplet.

Take care while recording the free-fall distance S_1. When looking through the eyepiece, the minor ticks represent 100 μ while the major ticks represent 1 mm. However, these measurements are magnified by a factor (1.875), so, a correction needs to be made.

Ultimately, the free-fall distance is calculated using the following equation:

$$S_1 = Ticks \times \frac{10^{-4}}{1.875} \; [m]$$

where S_1 is the fall distance, and *Ticks* refers to the "minor" tick marks over which the oil droplet fell. The free-fall velocity is calculated using the following equation:

$$v_1 = \frac{S_1}{t_1} \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$$

where v_1 is the free-fall velocity and t_1 is the free-fall time.

Use your measurements of v_1 and U to calculate r and Q found in Eqs. (5, 7).

A. Method 2

Again, we are not going to pursue Method 2 in our analysis of the Millikan oil-drop experiment.

B. Correction Factor

As mentioned before, the charge Q must be corrected to take into account the radius of each droplet. Based on Eq. 10, the corrected charge Q_C can be calculated. The constant b must be determined graphically and this is described in the Instruction Sheet (found on my website).

VI. SPECIAL CONSIDERATIONS

1. There is high voltage in this experiment, so be aware of this.

2. Use a level twice (in perpendicular directions) to make sure the Millikan Oil-Drop chamber is level before making measurements.

3. You will probably want to drape a dark cloth over your head and the experiment as you make the measurements. The experiment has its own light source to light up the oil drops as you view them in the apparatus

4. You can find the current "Surface Air Pressure" on our meteorology website: meteo.pr.erau.edu/ Go down the left side of this web page and "click" on *Current AC 1 rooftop weather*. You will find the surface air pressure in units of hPa (hectoPascals).