Homework Assignment #11

Chapter 5 The Schrödinger Equation

Modern Physics (3rd Edition) by Kenneth Krane

Due Date: Thursday, April 6, 2017

When the problem asks for mass, energy, and momentum, please write your answers in units of:

Mass $\rightarrow MeV/c^2$ not kilograms !!

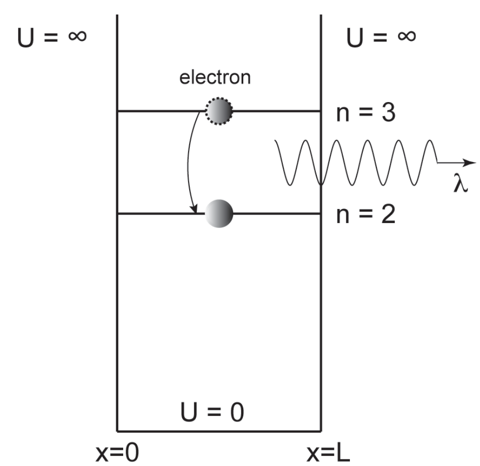
Momentum $\rightarrow MeV/c$ not kilograms·meters/sec !!

Energy $\rightarrow MeV, keV$ or eV not joules !!

unless otherwise specified. When you are asked for velocities, always quote your answers in units of "c," the speed of light.

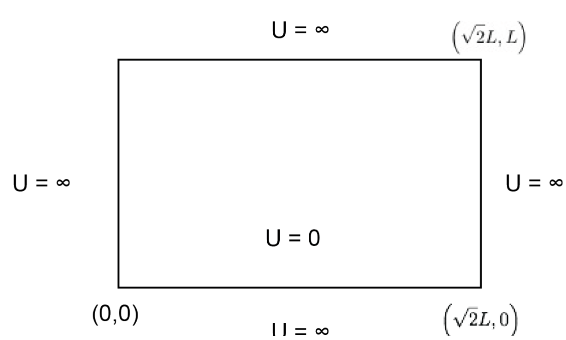
velocity = $\boldsymbol{\beta\, c}$

1. An electron is an infinitely deep potential well. It makes a transition from the $n = 3$ state to the $n = 2$ state, and during the process emits a photon having a wavelength of 656.28 nm. What is the width L of the well?



$L =$ ____________ nm

2. A particle is trapped in a 2-dimensional infinitely deep potential well. The well is rectangular as shown in the figure to the right.



a. Write out the equation for the energy states in terms of the quantum numbers n_x, n_y where the energy $E_o = \frac{\pi^2 \hbar^2}{2mL^2}$.

$E_{n_x,n_y} =$ ________________ E_o

b. At what quantum numbers do the lowest degenerate energy states occur?

What is the energy of these quantum states in terms of E_o? Provide your answer by filling in the following table:

	n_x	n_y	$E\ (E_o)$
1			
2			

3. A particle is in the ground state of a one-dimensional infinitely deep potential well of width L.

a. What is the probability of finding the particle in the region $(0 \le x \le 0.1\ L)$?

Probability = ____________ %

b. What is the probability of finding the particle in the region $(0.45\ L \le x \le 0.55\ L)$?

Probability = ____________ %

4. An ensemble of one-dimensional quantum harmonic oscillators is described by the following wave function:

$$\psi(x) = A\big(\ 4\,|0\rangle\ +\ 5\,|1\rangle + \ 2\sqrt{2}\,|2\rangle\,\big)$$

a. Calculate the normalization constant A for this wave function.

$A\ =$ ________

b. What is the mean energy (per oscillator) in this ensemble?

Calculate $\langle E\rangle = \ \big\langle\,\psi\,|\,E_{op}\,|\,\psi\rangle$ where $E_{op}\,|n\rangle\ =\ \left(n+\frac{1}{2}\right)\hbar\,\omega_o\,|n\rangle$ and

$|\,n\rangle$ are the orthonormal wave functions for the quantum harmonic oscillator.

$\langle E\rangle =$ ________ $\hbar\,\omega_o$

c. What is the probability for finding an oscillator with an energy of $\frac{3}{2}\ \hbar\,\omega_o$?

Probability = ________ %

5. An ensemble of one-dimensional infinitely deep wells (*i.e., quantum systems*) of width L are described by the following wave function:

$$\psi(x) = A\big(\ 4\,|1\rangle\ +\ 5\,|2\rangle + \ 2\sqrt{2}\,|3\rangle\,\big)$$

a. What is the normalization constant A ? $A =$ ________

b. What is the mean energy (per system) in this ensemble?

Calculate $\langle E\rangle = \ \big\langle\,\psi\,|\,E_{op}\,|\,\psi\rangle$ where $E_{op}\,|n\rangle = \frac{n^2\pi^2\hbar^2}{2mL^2}\,|n\rangle\ =\ n^2\,E_o\,|n\rangle$

where $|\,n\rangle$ are the orthonormal wave functions for the infinitely deep well.

$\langle E\rangle =$ ________ E_o