

Homework Assignment #11

Chapter 5 The Schrödinger Equation

Modern Physics (3rd Edition) by Kenneth Krane

Due Date: Tuesday, April 10, 2018

When the problem asks for mass, energy, and momentum, please write your answers in units of:

Mass $\rightarrow MeV/c^2$ not kilograms !!

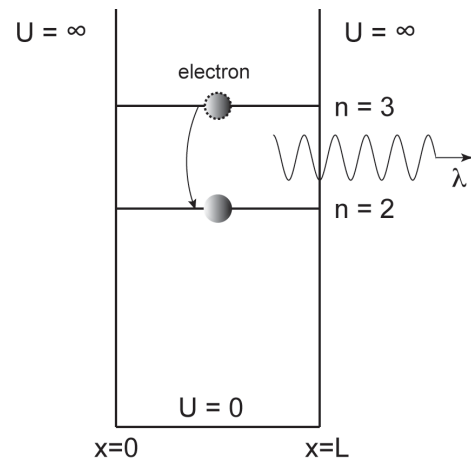
Momentum $\rightarrow MeV/c$ not kilograms·meters/sec !!

Energy $\rightarrow MeV, keV$ or eV not joules !!

unless otherwise specified. When you are asked for velocities, always quote your answers in units of "c," the speed of light.

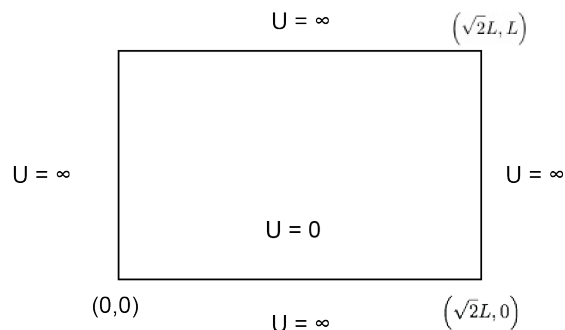
$$\text{velocity} = \beta c$$

1. An electron is in an infinitely deep potential well. It makes a transition from the $n = 3$ state to the $n = 2$ state, and during the process emits a photon having a wavelength of 656.28 nm. What is the width L of the well?



$$L = \text{_____} \text{ nm}$$

2. A particle is trapped in a 2-dimensional infinitely deep potential well. The well is rectangular as shown in the figure to the right.



- a. Write out the equation for the energy states in terms of the quantum numbers n_x , n_y where the energy $E_0 = \frac{\pi^2 \hbar^2}{2mL^2}$.

$$E_{n_x, n_y} = \text{_____} E_0$$

- b. At what quantum numbers do the lowest degenerate energy states occur?

What is the energy of these quantum states in terms of E_0 ? Provide your answer by filling in the following table:

	n_x	n_y	$E (E_0)$
1			
2			

3. A particle is in the ground state of a one-dimensional infinitely deep potential well of width L .

- a. What is the probability of finding the particle in the region $(0 \leq x \leq 0.1 L)$?

Probability = _____ %

- b. What is the probability of finding the particle in the region $(0.45 L \leq x \leq 0.55 L)$?

Probability = _____ %

4. An ensemble of one-dimensional quantum harmonic oscillators is described by the following wave function:

$$\psi(x) = A(4 |0\rangle + 5 |1\rangle + 2\sqrt{2} |2\rangle)$$

- a. Calculate the normalization constant A for this wave function.

$$A = \underline{\hspace{2cm}}$$

- b. What is the mean energy (per oscillator) in this ensemble?

Calculate $\langle E \rangle = \langle \psi | E_{op} | \psi \rangle$ where $E_{op} |n\rangle = \left(n + \frac{1}{2} \right) \hbar \omega_o |n\rangle$ and

$|n\rangle$ are the orthonormal wave functions for the quantum harmonic oscillator.

$$\langle E \rangle = \underline{\hspace{2cm}} \hbar \omega_o$$

- c. What is the probability for finding an oscillator with an energy of $\frac{3}{2} \hbar \omega_o$?

$$\text{Probability} = \underline{\hspace{2cm}} \%$$

5. An ensemble of one-dimensional infinitely deep wells (*i.e., quantum systems*) of width L are described by the following wave function:

$$\psi(x) = A(4 |1\rangle + 5 |2\rangle + 2\sqrt{2} |3\rangle)$$

- a. What is the normalization constant A ?

$$A = \underline{\hspace{2cm}}$$

- b. What is the mean energy (per system) in this ensemble?

Calculate $\langle E \rangle = \langle \psi | E_{op} | \psi \rangle$ where $E_{op} |n\rangle = \frac{n^2 \pi^2 \hbar^2}{2mL^2} |n\rangle = n^2 E_o |n\rangle$

where $|n\rangle$ are the orthonormal wave functions for the infinitely deep well.

$$\langle E \rangle = \underline{\hspace{2cm}} E_o$$