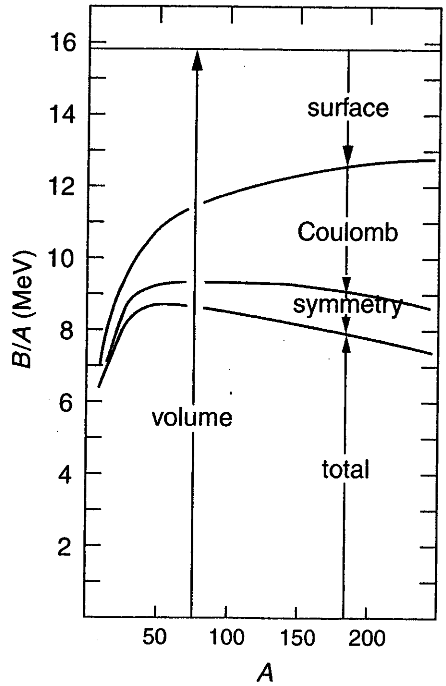
Figure 4.2 | Relative Importance of the Various Contributions to the Binding Energy Predicted by the Semiempirical Mass Formula



The free parameter, a_p, takes on a positive value for odd–odd nuclei indicating a decrease in stability, a negative value of even–even nuclei indicating an increase in stability and zero for odd A nuclei.

The total binding energy per nucleon as determined by the sum of the above terms and per nucleon is expressed

$$\frac{B}{A} = a_V - \frac{a_S}{A^{1/3}} - \frac{a_C Z(Z-1)}{A^{4/3}} - \frac{a_{sym}(A-2Z)^2}{A^2} - \frac{a_p}{A^{7/4}}. \qquad (4.10)$$

This can be compared with the experimental results as indicated in Figure 4.1 to obtain best fit values of the coefficients. The general shape of the curve is determined by the first four terms above. The relative importance of these terms is illustrated in Figure 4.2. The value of a_p is determined from a consideration of fluctuations in B/A as A changes from odd to even. This analysis gives the best fit values as

$$a_V = 15.5 \text{ MeV}$$

$$a_S = 16.8 \text{ MeV}$$

$$a_C = 0.72 \text{ MeV}$$

$$a_{sym} = 23.2 \text{ MeV}$$

$$a_p = +34 \text{ MeV} \qquad N, Z = \text{odd–odd}$$
$$0 \text{ MeV} \qquad A = \text{odd}$$
$$-34 \text{ MeV} \qquad N, Z = \text{even–even}. \tag{4.11}$$

Note that the value for a_C is in agreement with the analytical result calculated for a uniform charge distribution. As discussed in the next section, these values can be used to describe the stability of certain nuclides.

4.3 BETA STABILITY

From the development given above the total atomic mass can be determined to be

$$m = (A - Z)m_n + Z(m_p + m_e) - \frac{a_V A}{c^2} + \frac{a_S A^{2/3}}{c^2} + \frac{a_C Z(Z-1)}{A^{1/3}c^2} + \frac{a_{sym}(A-2Z)^2}{Ac^2} + \frac{a_p}{A^{3/4}c^2} \tag{4.12}$$

where N is written as $A - Z$ and the electronic binding energy is ignored. It is interesting to consider the Z dependence of this expression for a constant value of A. This is equivalent to examining the behavior of isobars. Collecting together powers of Z, it is seen that the above expression is a quadratic. The simplest case to consider is for nuclei with odd A since the pairing term is zero. An example of plotting m versus Z at constant A is shown in Figure 4.3 for $A = 135$.

Figure 4.3 | Mass Parabola for $A = 135$ Showing One Stable Nuclide (as Expected for Odd A) with $Z = 56$

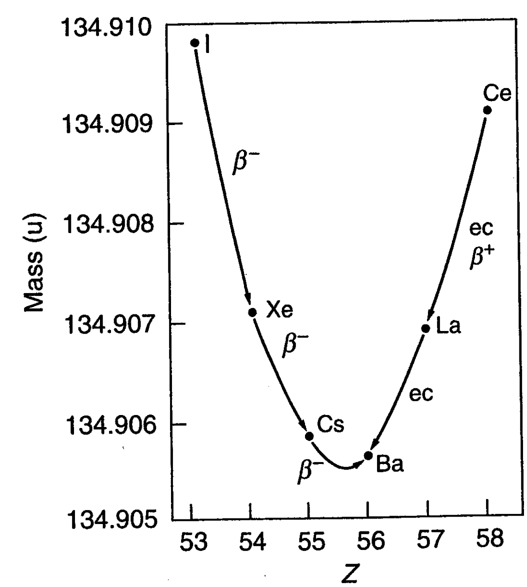


Table 4.1 | Properties of Nuclides with $A = 135$ (ec = electron capture)

Element	N	Z	Mass (u)	Lifetime	Decay Mode	Daughter	Decay Energy (MeV)
I	82	53	134.909823	9.6 h	β^-	^{135}Xe	2.51
Xe	81	54	134.907130	13.1 h	β^-	^{135}Cs	1.16
Cs	80	55	134.905885	4.3×10^6 y	β^-	^{135}Ba	0.21
Ba	79	56	134.905665	stable	—	—	—
La	78	57	134.906953	28.0 h	ec	^{135}Ba	1.20
Ce	77	58	134.909117	25.3 h	ec, β^+	^{135}La	2.02

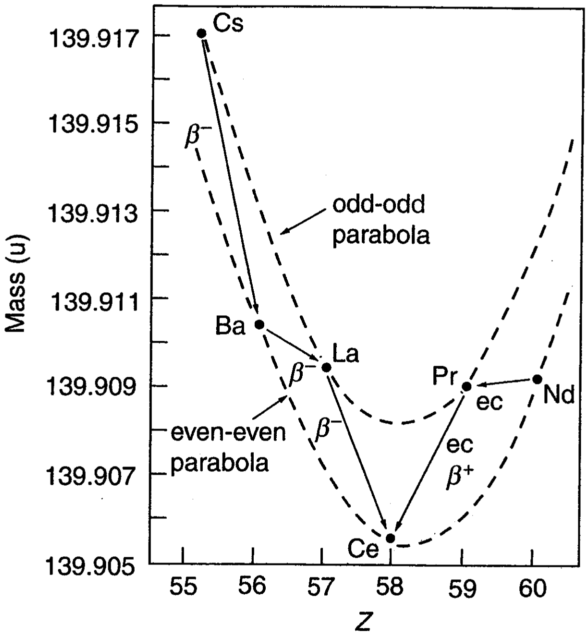
These β-decay processes, as well as electron capture, which is equivalent to β^+ decay, will be considered in detail in Chapter 9. Table 4.1 gives some of the relevant properties for the nuclides shown in Figure 4.3. A general observation can be made concerning the lifetimes of the nuclides described in the table. As the nuclides decay towards ^{135}Ba from either side the lifetimes become longer. This is directly related to the decreasing difference in mass between the parent and daughter nucleus. This same feature is seen in subsequent decays described in this chapter.

On the basis of Figure 4.3 it is expected that for a given value of odd A there should be uniquely one β stable nuclide. Since odd A can result from N and Z odd–even or even–odd we would expect approximately equal numbers of these two types of nuclei to exist. This is in agreement with Table 3.2.

The situation for even A, which occurs for odd–odd or even–even nuclei, is much more complex because the pairing term is nonzero. In general we expect two parabolas for m as a function of Z, one shifted up (the odd–odd parabola) and one shifted down (the even–even parabola) as illustrated in Figure 4.4 for $A = 140$. The stable nucleus again occurs for the minimum value of mass and the figure shows that as nuclei with too few or too many protons decay by β-decay processes they alternate from the odd–odd parabola to the even–even parabola. For the case shown ^{140}Ce with $Z = 58$ lies very close to the minimum in m and represents the β stable nuclide with $A = 140$. Properties of the decay process shown in the figure are given in Table 4.2.

Another situation for an even A nucleus ($A = 128$) is illustrated in Figure 4.5. Here there are two stable nuclei on the even–even parabola; ^{128}Te and ^{128}Xe. ^{128}I can decay by either β^- decay to ^{128}Xe or by β^+ decay to ^{128}Te. This means that it becomes more stable either by converting a neutron to a proton or a proton to a neutron. This can be understood on the basis of the semiempirical mass formula because either process will change an odd–odd nucleus to an even–even nucleus.

Figure 4.4 | Mass Parabola for $A = 140$ Showing One Stable Nuclide with $Z = 58$



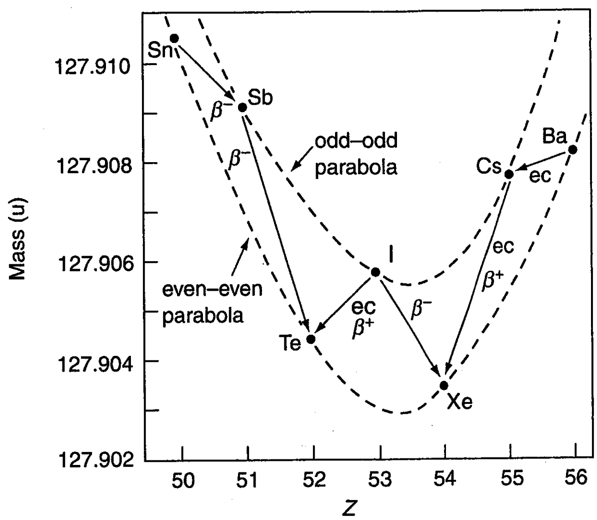
However, the real implications of this pairing behavior will become obvious in the next chapter. Relevant nuclear properties are given in Table 4.3.

A third possibility for an even A nucleus is shown in Figure 4.6 ($A = 130$). Here there are three stable nuclei on the even–even parabola; ^{130}Te, ^{130}Xe, and ^{130}Ba. In this case ^{130}Cs can decay by either β^- or β^+ decay to ^{130}Xe or

Table 4.2 | Properties of Nuclides with $A = 140$ (ec = electron capture)

Element	N	Z	Mass (u)	Lifetime	Decay Mode	Daughter	Decay Energy (MeV)
Cs	85	55	139.917338	95 s	β^-	^{140}Ba	6.29
Ba	84	56	139.910518	18.3 d	β^-	^{140}La	1.03
La	83	57	139.909471	58.0 h	β^-	^{140}Ce	3.76
Ce	82	58	139.905433	stable	—	—	—
Pr	81	59	139.909071	4.9 m	ec, β^+	^{140}Ce	3.39
Nd	80	60	139.909036	4.75 d	ec	^{140}Pr	0.22

Figure 4.5 | Mass Parabola for $A = 128$ Showing Two Stable Nuclides with $Z = 52$ and 54

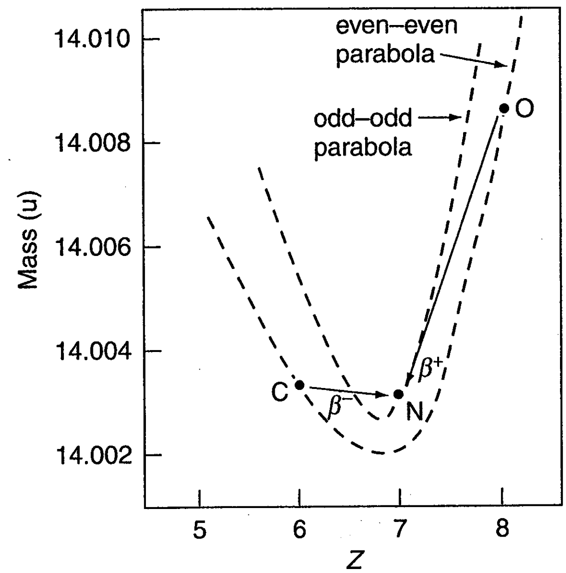


^{130}Ba, respectively. ^{130}I decays by β^- decay to ^{130}Xe. Although it is energetically favorable for ^{130}I to decay to ^{130}Te, this process has not been observed. Relevant nuclear properties are given in Table 4.4. The stability of nuclides such as ^{130}Te, which are at a local minimum on the mass curve will be discussed further in Chapter 9 in the context of double β-decay.

Table 4.3 | Properties of Nuclides with $A = 128$ (ec = electron capture)

Element	N	Z	Mass (u)	Lifetime	Decay Mode	Daughter	Decay Energy (MeV)
Sn	78	50	127.910467	85 m	β^-	^{128}Xe	1.30
Sb	77	51	127.909072	15.6 m	β^-	^{128}Cs	4.29
Te	76	52	127.904463	stable	—	—	—
I	75	53	127.905810	36 m	ec, β^+ β^-	^{128}Te ^{128}Xe	1.26 2.12
Xe	74	54	127.903531	stable	—	—	—
Cs	73	55	127.907762	5.5 m	ec, β^+	^{128}Xe	3.94
Ba	72	56	127.908237	3.46 d	ec	^{128}Cs	0.44

Figure 4.7 | Mass Parabola for $A = 14$ Showing the Existence of a Stable Odd–Odd Nuclide



odd–odd nuclei could be stable, since the odd–odd nucleus can decay to the even–even nuclei on either side. If we examine the shape of the mass parabola as given by equation (4.12), we learn that the parabola becomes narrower as A decreases. Thus for small A we can have the situation as shown in Figure 4.7 for $A = 14$. Here the sides of the parabolas are sufficiently steep that the minimum in the odd–odd parabola lies below the adjacent points on the even–even parabola and the odd–odd nucleus ^{14}N is the stable $A = 14$ nuclide. Other situations can exist, for example ^{2}H, where β-decay cannot occur because the daughter nucleus does not form a bound state.

The predictions of the semiempirical mass formula can be viewed in the context of the distribution of stable nuclides as was shown in Figure 3.1. In this figure, each particular point in N-Z space corresponds to a particular value of A. Constant A isobars are represented in the figure by lines parallel to the $A = 100$ line shown. Thus in a three-dimensional plot with Figure 3.1 as the xy plane and mass plotted on the z-axis, mass parabolas for constant A lines would form a parabolic surface with the minimum following the stability line in the N-Z plane. This is referred to as the β-stability valley.

4.4 NUCLEON SEPARATION ENERGIES

A measure of nuclear stability that will be discussed in some detail in the next chapter, is the energy required to remove one nucleon from the nucleus. This is not the same for neutrons and protons. We can define two quantities, S_n and S_p, the neutron separation energy and the proton separation energy, respectively.