Binding Energy and the Liquid Drop Model
Semiemperical Mass Formula

1 Nuclear Binding Energy

- The nuclear binding energy, B, is the minimum energy required to separate a nucleus into its component neutrons and protons. The mass of the nucleus can be written as:

$$m_N = Nm_n + Zm_p - \frac{B}{c^2}$$

- The nuclear stability increases as B increases.
- The nuclear masses are usually not measured directly. Instead the atomic mass of a neutral atom is measured:

$$m = Nm_n + Z\left(m_p + m_e\right) - \frac{B}{c^2} - \frac{b}{c^2}$$

  where b is the total binding energy associated with all the atomic electrons.

$$b = 20.8 \times Z^{7/3} \qquad \text{(eV)}$$

- Tables of nuclides sometimes include the "mass excess"

$$\Delta = (m - Au)c^2$$

  where A is the number of nucleons and u is the mass of one atomic mass unit. $1u = 1.6605 \times 10^{-27}$ kg
- The total nuclear binding energy shows an approximately linear relationship to A. What does this mean? It means that to first order, each nucleon is bound to the nucleus with the same energy regardless of the size of the nucleus. In other words, B/A should be $\sim$constant.
- Figure 4.1 in your book. The Binding Energy per Nucleon as a Function of Mass Number, A.

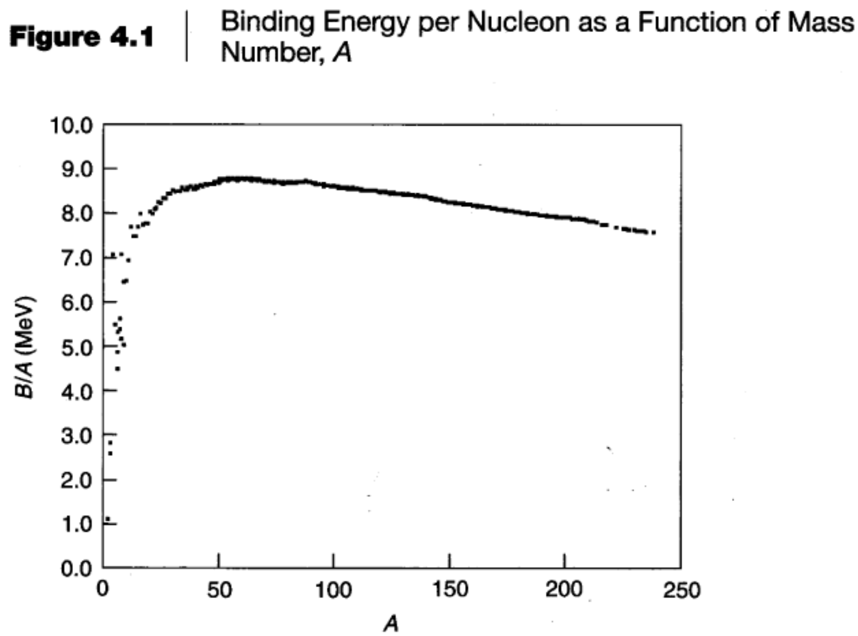

Figure 1: Binding Energy Per Nucleon

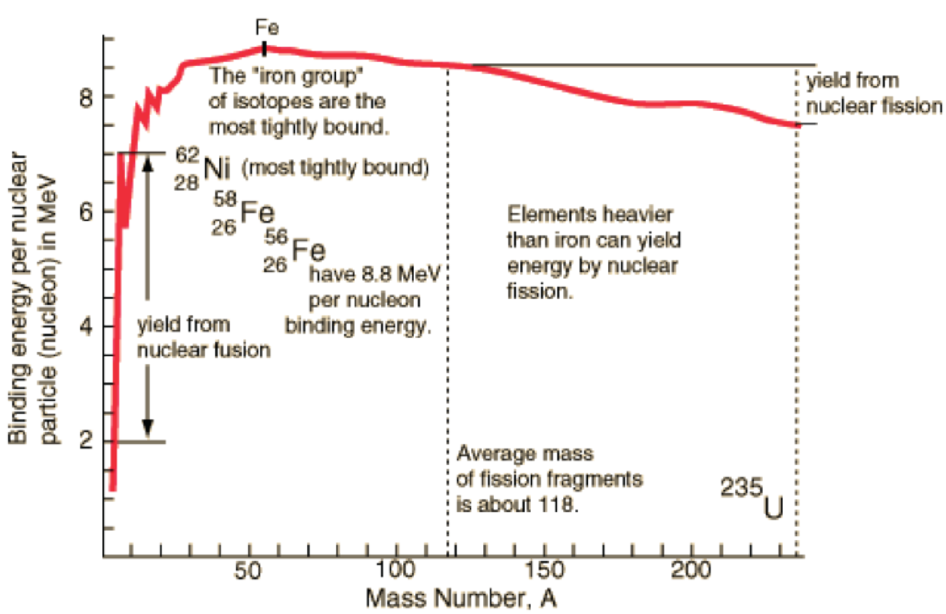

Figure 2: Binding Energy Per Nucleon describing fission and fusion

2 The Liquid Drop Model

- Much is known about the nucleon-nucleon interaction from experimental measurements.
- Protons and neutrons interact with each other in a very similar way.
- The interaction (the strong force) is both strong and short range.
- The nucleon-nucleon interactions are spin dependent.
- The nucleon-nucleon interactions have both central and non-central components.
- A thorough analysis of the nucleus can be done, but it is very complicated. The most salient features of the nucleus can be found in the construction of two simple models: The liquid-drop model, and the shell model.

2.1 The liquid drop model

- The liquid drop model describes the nuclear binding energy as a function of the number of neutrons and protons.
- It is semiemperical in nature, and will lead to what's called the "semiemperical mass formula."

2.2 The semiemperical mass formula

The binding energy per nucleon is parameterized in the following manner:

$$\frac{B}{A} = a_V - \frac{a_S}{A^{1/3}} - \frac{a_C Z(Z-1)}{A^{4/3}} - \frac{a_{\mathrm{sym}}(A-2Z)^2}{A^2} - \frac{a_p}{A^{7/4}} \tag{1}$$

- $a_V = 15.5$ MeV the volume term
- $a_S = 16.8$ MeV the surface term
- $a_C = 0.72$ MeV the coulomb term

- $a_{sym} = 23.2$ MeV   the symmetry term   Favors $N = Z$, but becomes less important as A becomes large.
- a_p the pairing term

$$a_p = \begin{cases} +34 \text{ MeV} & N, Z = odd - odd \\ 0 \text{ MeV} & A = odd \\ -34 \text{ MeV} & N, Z = even - even \end{cases}$$

Figure 4.2 | Relative Importance of the Various Contributions to the Binding Energy Predicted by the Semiempirical Mass Formula

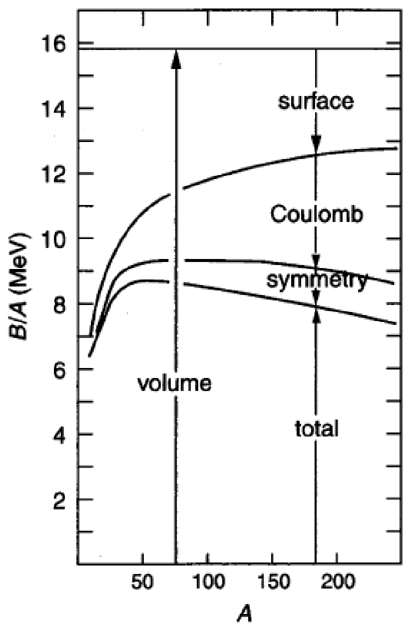

Figure 3: Contributions to the Binding Energy Per Nucleon

2.3 Beta Stability

Now that we know how to calculate the total binding energy B (Eq. 1 above), we can calculate the total atomic mass m. Keep in mind that minimizing the mass corresponds to maximizing nuclear stability.

$$m = (A-Z)m_n + Z(m_p + m_e) - \frac{a_V A}{c^2} + \frac{a_S A^{2/3}}{c^2} + \frac{a_C Z(Z-1)}{A^{1/3}\,c^2} + \frac{a_{\text{sym}}(A-2Z)^2}{A\,c^2} + \frac{a_p}{A^{3/4}\,c^2}$$

Figure 4.3 | Mass Parabola for $A = 135$ Showing One Stable Nuclide (as Expected for Odd A) with $Z = 56$

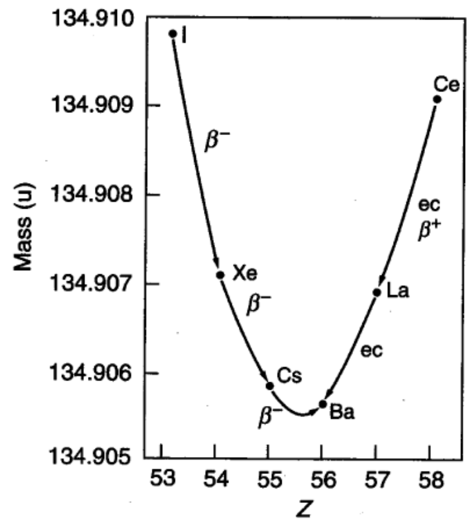

Figure 4: The nuclear mass parabola as a function of Z for a fixed value of A.

The total atomic mass m can now be plotted as a function Z for a fixed value of A resulting in a parabolic curve whose minimum is close to Z_{min}. Taking the derivative dm/dZ and setting equal to zero gives the value of Z which minimizes the mass, Z_{min}.

$$Z_{min} = \frac{(m_n - m_p - m_e)c^2 + a_C\, A^{-1/3} + 4a_{sym}}{2a_C\, A^{-1/3} + 8a_{sym}\, A^{-1}}$$

The Z value closest to Z_{min} is considered to be the most stable nuclide.

2.3.1 Even values of A

When plotting m vs. Z for even A values, two parabolas will appear. The upper one is the odd-odd parabola, and the lower one is the even-even parabola.

2.3.2 Odd values of A

When plotting m vs. Z for odd A values, only one parabola will appear. The one parabola results from $a_p = 0$. The one parabola represents both even-odd and odd-even when counting protons and neutrons.

2.4 Nucleon Separation Energies

The energy required to remove a neutron or a proton from a nucleus is not the same.

S_n = the energy required to remove a neutron
S_p = the energy required to remove a proton

2.4.1 The removal of a neutron

The removal of a neutron from a nucleus correspond to the following process:

$$ {}^{A}_{Z}X^{N} \;\rightarrow\; {}^{A-1}_{Z}X^{N-1} + n \tag{2}$$

Assuming this process is *endothermic*, the separation energy S_n required to remove the neutron would be:

$$ S_n = \left[m\left({}^{A-1}_{Z}X^{N-1}\right) + m_n - m\left({}^{A}_{Z}X^{N}\right)\right] c^2 \tag{3}$$

There is no change in the number of electrons.

2.4.2 The removal of a proton

$$ {}^{A}_{Z}X^{N} \;\rightarrow\; {}^{A-1}_{Z-1}Y^{N} + p \tag{4}$$

$$ S_p = \left[m\left({}^{A-1}_{Z-1}Y^{N}\right) + m_p + m_e - m\left({}^{A}_{Z}X^{N}\right)\right] c^2 \tag{5}$$

where the electron mass is included to allow for the use of atomic masses (Appendix B of “The Physics of Nuclei and Particles”).

See Table 4.5 for the Binding Energies and Nucleon Separation Energies for Light Nuclei.

See Figures 4.8 and 4.9 for Mass Parabolas showing the Neutron Separation Energies and Proton Separation Energies.