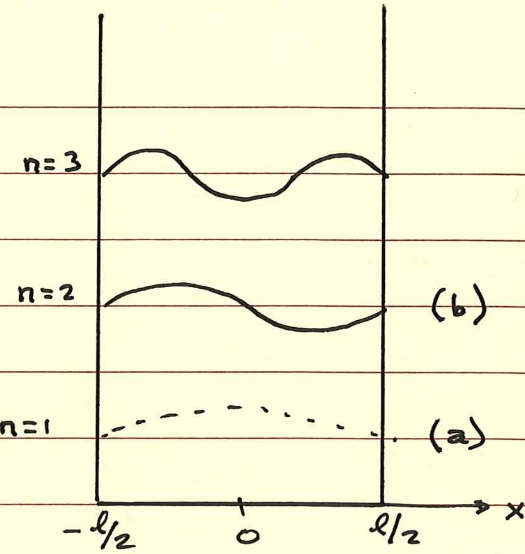
DATE	
TOPIC	Time Depend. Perturbation

"Rocking Floor Perturbation"

$$H'(x,t) = V(x,t) = V_0 \frac{x}{\ell} \frac{t}{\tau} e^{\frac{-t^2}{2\tau^2}}$$



$$\psi_1(x) = \sqrt{\frac{2}{\ell}} \cos\left(\frac{\pi x}{\ell}\right)$$
$$\psi_2(x) = \sqrt{\frac{2}{\ell}} \sin\left(\frac{2\pi x}{\ell}\right)$$
$$\psi_3(x) = \sqrt{\frac{2}{\ell}} \cos\left(\frac{3\pi x}{L}\right)$$

$$\boxed{\dot{c}_b(t) = \frac{-i}{\hbar} c_a(0) H'_{ba} e^{+i\omega_0 t}}$$

$c_a(0) = 1$ and $c_b(-\infty) = 0$

$$\omega_0 = \frac{E_b - E_a}{\hbar}$$

$$\dot{c}_b(t) = \frac{-i}{\hbar} (1) V_{ba} \frac{t}{\tau} e^{\frac{-t^2}{2\tau^2}} e^{i\omega_0 t}$$

Integrate $-\infty \leq t \leq +\infty$

where $V_{ba} = V_0 \langle \psi_b | \frac{x}{\ell} | \psi_a \rangle$

$$c_b(t) = \frac{-i}{\hbar} V_0 \langle \psi_b | \frac{x}{\ell} | \psi_a \rangle \int_{-\infty}^{\infty} \frac{t}{\tau} e^{\frac{-t^2}{2\tau^2}} e^{i\omega_0 t} dt + \overset{0}{\overset{\shortparallel}{c_b(-\infty)}}$$

Constant of Integration

$$c_b(t) = \frac{-i}{\hbar} V_0 \langle \psi_b | \frac{x}{\ell} | \psi_a \rangle \, 2i \int_0^{\infty} \frac{t}{\tau} e^{\frac{-t^2}{2\tau^2}} \sin(\omega_0 t)\, dt$$

$$c_b(t) = \frac{2V_0}{\hbar} \langle \psi_b | \frac{x}{\ell} | \psi_a \rangle \int_0^{\infty} \frac{t}{\tau} e^{\frac{-t^2}{2\tau^2}} \sin(\omega_0 t)\, dt$$

$$c_b(\tau) = \frac{2V_0}{\hbar} \left(\frac{16}{9\pi^2}\right) \left(\sqrt{\frac{\pi}{2}}\, \tau^2 \omega_0 e^{\frac{-\omega_0^2 \tau^2}{2}}\right)$$

$$\boxed{c_b(\tau) = \frac{16\sqrt{2}\, V_0 \omega_0 \tau^2 e^{-\frac{\omega_0^2 \tau^2}{2}}}{9\pi^{3/2} \hbar}}$$

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"Rocking Floor Perturbation"

What τ will result in the maximum transition probability ?

$$|c_b(\tau)|^2 = \frac{512}{81} \frac{V_o^2 \omega_o^2 \tau^4 e^{-\omega_o^2 \tau^2}}{\pi^3 \hbar^2}$$

$$\frac{\partial}{\partial \tau} |c_b(\tau)|^2 = 0 \quad \Rightarrow \quad \tau = 0, \; -\frac{\sqrt{2}}{\omega_o}, \; +\frac{\sqrt{2}}{\omega_o}$$

$$\boxed{\text{Max. Probability} = |c_b(\tau_{max})|^2 = \frac{2048}{81} \frac{V_o^2 e^{-2}}{\pi^3 \omega_o^2 \hbar^2}}$$

$$\tau \rightarrow +\frac{\sqrt{2}}{\omega_o} = \frac{\sqrt{2}\,\hbar}{3E_1}$$

$$\omega_o \rightarrow \frac{E_2 - E_1}{\hbar} = \frac{3E_1}{\hbar}$$

Assume $V_o \approx \frac{1}{2} E_1$

Then, $$|c_b(\tau_{max})|^2 = \frac{2^9}{3^6} \frac{e^{-2}}{\pi^3} = 3.07 \times 10^{-3}$$

$$\boxed{\underset{(1 \rightarrow 2)}{\text{Prob}\,(a \rightarrow b)} = |c_b(\tau_{max})|^2 = 0.31\%}$$

if $V_o = \frac{1}{2} E_1$
and $\tau = \sqrt{2}/\omega_o$