2^{nd} Order Perturbation Theory (Energy only)

Starting with Eq. 2 (back 4 pages) where we compared terms in λ^2, we have

Take the inner $\langle \psi_n^0|$

$$H'\psi_n^1 + H^0\psi_n^2 = E_n^2\psi_n^0 + E_n^1\psi_n^1 + E_n^0\psi_n^2$$

$$\langle \psi_n^0|H'|\psi_n^1\rangle + \langle \psi_n^0|H^0|\psi_n^2\rangle = E_n^2\underbrace{\langle \psi_n^0|\psi_n^0\rangle}_{=1} + E_n^1\cancel{\langle \psi_n^0|\psi_n^1\rangle} + E_n^0\cancel{\langle \psi_n^0|\psi_n^2\rangle}$$

$$\langle H^0\psi_n^0|\psi_n^2\rangle = E_n^0\langle \psi_n^0|\psi_n^2\rangle$$

So, $$E_n^2 = \langle \psi_n^0|H'|\psi_n^1\rangle - E_n^1\langle \psi_n^0|\psi_n^1\rangle$$

However, $$\langle \psi_n^0|\psi_n^1\rangle = \sum_{m\neq n} c_m^{(n)}\langle \psi_n^0|\psi_m^0\rangle = 0$$

because $m \neq n$

So, $$E_n^2 = \langle \psi_n^0|H'|\psi_n^1\rangle = \sum_{m\neq n} c_m^{(n)}\langle \psi_n^0|H'|\psi_m^0\rangle$$

but $$c_m^{(n)} = \frac{\langle \psi_m^0|H'|\psi_n^0\rangle}{E_n^0 - E_m^0}$$

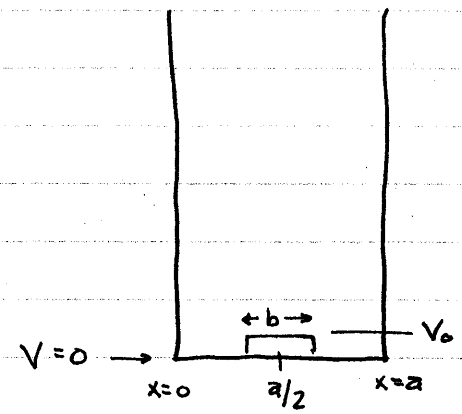
So, $$\boxed{E_n^2 = \sum_{m\neq n}\frac{|\langle \psi_m^0|H'|\psi_n^0\rangle|^2}{E_n^0 - E_m^0}}$$

The 2^{nd} order correction to the energy.

Homework Problem:

$$\psi_n^{\circ}(x) = \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a}$$

Infinitely deep potential well



a.) Find the 1st and 2nd order corrections to the ground state energy.

$$E = E_1^{\circ} + E_1^{1} + E_1^{2} = ?$$

b.) Plot $E^{1}_{n=1}$, the first order correction as a function of b.

c.) Plot $\psi_{n=1} = \psi_1^{\circ} + \psi_1^{1}$ ← 1st order correction including ~~for~~ the first 10 non-terms terms of $\psi_1^{1} = \sum\limits_{m\neq 1} \dfrac{\langle \psi_m^{\circ} | H' | \psi_m^{\circ} \rangle}{E_1^{\circ} - E_m^{\circ}}$

Note: $E_n^{\circ} = \dfrac{n^2\pi^2\hbar^2}{2ma^2}$ Let $V_0 = \alpha \dfrac{\pi^2\hbar^2}{2ma^2}$