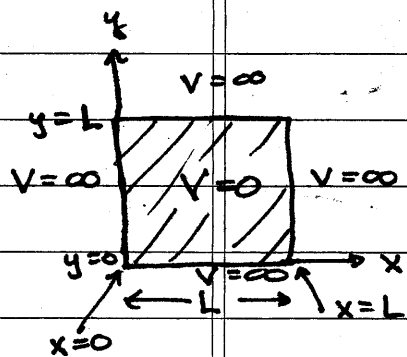
DATE	
TOPIC	

PS 405 Atomic & Nuclear Physics



Example: A particle in a 2-dimensional, infinitely deep potential well is described by the following wave function:

$$\psi(x,y) = 5\,|1,1\rangle + 4\,|1,2\rangle + \sqrt{8}\,|2,2\rangle$$

where $|n_x, n_y\rangle = \frac{2}{L}\sin\left(\frac{n_x \pi x}{L}\right)\sin\left(\frac{n_y \pi y}{L}\right)$

a.) Normalize this wavefunction.

$$\psi_{norm} = A\,\psi(x,y) \qquad \langle \psi_{norm} | \psi_{norm} \rangle = 1$$

$$\langle \psi_{norm} | \psi_{norm} \rangle = \langle A\,\psi(x,y) | A\,\psi(x,y) \rangle = |A|^2 \langle \psi(x,y) | \psi(x,y) \rangle$$

$$= |A|^2 (25 + 16 + 8) = 49|A|^2 = 1$$

$$A = \frac{1}{7}$$

$$\boxed{\psi_{norm} = A\,\psi(x,y) = \frac{5}{7}|1,1\rangle + \frac{4}{7}|1,2\rangle + \frac{\sqrt{8}}{7}|2,2\rangle}$$

b.) Find the mean energy for an ensemble of systems described by this wave function. $E_{n_x n_y} = \left[\frac{\pi^2\hbar^2}{2mL^2}\right](n_x^2 + n_y^2)$

$$= E_0\,(n_x^2 + n_y^2)$$

$$\langle E \rangle = \langle \psi_{norm} | E | \psi_{norm} \rangle$$

$$= \left[\frac{5}{7}\langle 1,1| + \frac{4}{7}\langle 1,2| + \frac{\sqrt{8}}{7}\langle 2,2|\right] E \left[\frac{5}{7}|1,1\rangle + \frac{4}{7}|1,2\rangle + \frac{\sqrt{8}}{7}|2,2\rangle\right]$$

$$\langle E \rangle = \frac{25}{49}E_{1,1} + \frac{16}{49}E_{1,2} + \frac{8}{49}E_{2,2}$$

$$\langle E \rangle = \frac{25}{49}(2E_0) + \frac{16}{49}(5E_0) + \frac{8}{49}(8E_0) = \frac{194}{49}E_0 = \underline{\underline{3.96\,E_0}}$$