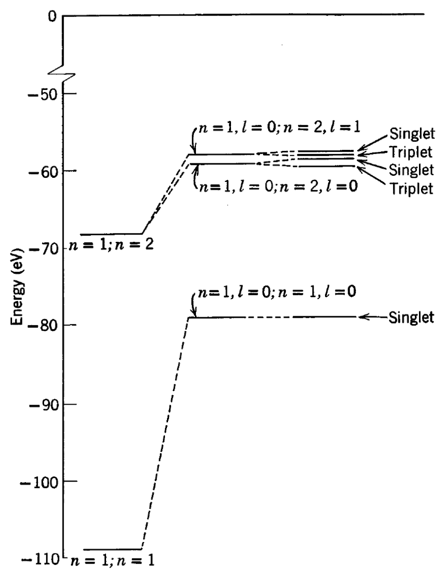
Homework 5

Due: Monday, March 6 2017

1. The figure to the right shows the low-lying energy levels of helium. The energy levels in the left column show the energy of the two-electron system with no Coulomb interaction between the electrons (i.e., $V_{ee} = 0$). The energy levels in the center column show the location of the two-electron system energy levels if the Coulomb interaction is turned on (i.e., $V_{ee} \neq 0$). The energy levels in the right column include the Coulomb interaction and the exchange force for the two-electron system. Write down the spectroscopic notation for the 3 singlet states and 2 triplet states, the energy levels shown in the right-hand column.



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2. We worked out (in class) the kinetic energy of the degenerate electron gas in a white dwarf star. We also calculated its gravitational potential energy. We noticed that the Fermi energy of the electron gas was relativistic, but we had assumed that the kinetic energy was non-relativistic in the calculation.

   a. Repeat this calculation assuming that the electrons are ultra-relativistic, (i.e., $KE = pc = \hbar kc$). Find the kinetic energy of the degenerate electron gas assuming the ultra-relativistic approximation.

   b. What is the number of nucleons N where the kinetic energy of the degenerate electron gas balances the gravitational potential energy of the white dwarf? This value is called N_C, the Chandrasekhar limit.

   N = ________________ nucleons

   c. What is the mass of the star at the Chandrasekhar limit in units of solar masses, $M_\odot$.

   M = __________ $M_\odot$

Neutron Star

3. If the mass of the white dwarf exceeds the mass calculated in (2. c), and if the conditions are right, the star will undergo inverse beta decay, and the remaining electrons and protons will convert to neutrons and neutrinos, with the neutrinos leaving the star. The neutron degeneracy pressure will stabilize the collapse, and the star will become a neutron star.

   a. Repeat the calculation we did in class for the white dwarf star, but for the neutron star, to determine the radius of the neutron star. Assume that the neutrons are non-relativistic in this calculation ($K = \frac{(\hbar k)^2}{2\, m_N}$) where m_N is the mass of a neutron, and that the number of neutrons in the neutron star is the number of neutrons at the Chandrasekhar limit.

   $R_{neutron\ star} =$ ______________ km

   b. Calculate the Fermi energy of the neutron star (at the Chandrasekhar limit) to confirm whether or not the non-relativistic assumption was a good one.

   $E_F =$ ______________ MeV

   Is this considered relativistic? Show why or why not.