Homework 11 PS405

Due: Monday, November 28, 2016

Some of the problems are from "The Physics of Nuclei and Particles," by Richard A Dunlap

Chapter 4

4.2 Modified

Use Equation 4.10 to calculate the binding energy/nucleon (B/A) for ${}^{4}_{2}He$, ${}^{56}_{26}Fe$, and ${}^{197}_{79}Au$. Compare these to the Wolfram Database for Isotope Data.*

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IsotopeData[{2, 4}, "BindingEnergy"]
   .   .   .   etc.
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Determine the percent difference between the *calculated* B/A (using the semiempirical mass formula) and the *database* B/A.

	${}^{4}_{2}He$	${}^{56}_{26}Fe$	${}^{197}_{79}Au$
$e =$	7.69508	8.83574	7.83299
$f =$	7.073915	8.790323	7.915661
$\frac{f-e}{f} =$	8.78%	= 0.517%	= 1.04%

4.4 (a) Using measured atomic masses *m* calculate the binding energies of ${}^{13}C$ and ${}^{13}N$. These nuclei are referred to as mirror nuclei.

Use Eq. 4.2 to calculate B (ignore *b* the binding energy of the electrons).

$B({}^{13}_{6}C) =$ 97.109 MeV

$B({}^{13}_{7}N) =$ 94.106 MeV

(b) On the basis of the liquid drop model, describe the reason(s) for the differences observed in part (a). The only difference is due to the Coulomb term.

$$B(Z=6) - B(Z=7) = -\frac{a_c}{A^{1/3}}\left[6\cdot 5 - 7\cdot 6\right] = -\frac{a_c}{A^{1/3}}[-12] = 3.0028 \text{ MeV}$$

(c) From these results, calculate $A^{1/3}$ from the coulomb term and calculate the radius of the ${}^{13}_{6}C$ nucleus.

$$A^{1/3} = \frac{12\, a_c}{3.0028} = \frac{12\,(0.72 \text{ MeV})}{3.0028 \text{ MeV}} = 2.8773$$

$$\boxed{A^{1/3} = 2.8773}$$

$R_o =$ 3.45 ~~MeV~~ fm

Eq. 3.23 $\rightarrow$ $R_o = (1.2 \text{ fm})\, A^{1/3}$

$R_o = 3.45$ fm

4.8 Modified

Using the semiempirical mass formula, calculate the most stable value of Z for nuclei having the following number of nucleons: A = 10, 50, 100, 200.

$Z =$ 4 $\quad Z =$ 22 $\quad Z =$ 44 $\quad Z =$ 80

Problem 4

The ground-state wave-function of a lepton of mass m in a Coulomb potential $-\frac{Ze^2}{4\pi\epsilon_o r}$ is

$$\psi(r) = \frac{1}{\sqrt{\pi}}\left(\frac{Z}{a}\right)^{3/2} e^{-Zr/a}$$

where a is the Bohr radius ($\hbar/(mac)$), and the corresponding binding energy E is $Z^2\hbar^2/2ma^2$. The finite size of the nucleus modifies the Coulomb energy for $r < R$, the nuclear radius, by adding a term of the approximate form

$$V(r) = -\frac{Ze^2}{4\pi\epsilon_o R}\left[\frac{3}{2} - \frac{r^2}{2R^2} - \frac{R}{r}\right]$$

(a) Show that the volume integral ($0 \to R$) of this potential energy is $\int V(r)d^3r = \frac{Ze^2R^2}{10\epsilon_0}$.

$$= 4\pi \frac{-Ze^2}{4\pi\epsilon_o R}\int_0^R r^2\left(\frac{3}{2} - \frac{r^2}{2R^2} - \frac{R}{r}\right)dr = +\frac{Ze^2}{4\pi\epsilon_o R}\left[\frac{2\pi R^3}{5}\right] = +\frac{Ze^2R^2}{10\,\epsilon_o}$$

(b) Show that the first-order correction to the binding energy due to this term, $\Delta E = \int \psi^*(r)V(r)\psi(r)d^3r$, is

$$= \psi^*(0)\psi(0)\int_0^R V(r)\,d^3r \approx$$

$$\Delta E \approx \frac{e^2}{10\pi\epsilon_o}\frac{Z^4R^2}{a^3}$$

(Note that the lepton wave-function can be taken to be constant over nuclear dimensions.)

(c) For the nucleus $^{66}_{30}Zn$ show that: $\frac{\Delta E}{E} = \frac{e^2 m R^2 Z^2}{5\pi a\,\epsilon_o \hbar^2} = \left(\frac{e^2}{4\pi\epsilon_o\hbar c}\right)\frac{Z^2\alpha R^2 m^2 c^2 4}{5\pi\epsilon_o\hbar^2}$

$$\frac{\Delta E}{E} = \frac{Z^2\alpha^2R^2(mc^2)^2}{(\hbar c)^2}\,\frac{4}{5}$$

$\frac{\Delta E}{E} \approx 5\times10^{-6}$ for electrons, $= 6.07\times10^{-6}$

$\frac{\Delta E}{E} \approx 0.2$ for muons. $= 0.26$

Problem 5

In an experiment using 14 g of selenium containing 97% by weight of $^{82}_{34}Se$, 35 events associated with the double β-decay

$$^{82}_{34}Se \to {}^{82}_{36}Kr + 2e^- + 2\bar{\nu}_e$$

# of selenium $^{82}_{34}Se$ atoms $= \frac{0.97\,(6.022\times10^{23})\;14\,g}{82\;g/mol}$

$\# = 9.973\times10^{22}$ atoms

$$\text{Rate} = \frac{35 \text{ decays}}{7960 \text{ hrs}\left(\frac{3600\,s}{hr}\right) 9.973\times10^{22}\text{ atoms }(0.062)} = 1.975\times10^{-28}\,sec^{-1}$$

$$\tau = \frac{1}{\text{Rate}} = 5.06\times10^{27}\,sec = 1.602\times10^{20}\text{ yrs}$$

$t_{1/2} = \tau \ln 2 = 1.11\times10^{20}$ yrs compared to 0.96×10^{20} years