Radioactive Decay– Measuring the half-life $t_{1/2}$

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The purpose of this experiment is to measure the half-life of barium-137 (^{137m}Ba). More precisely, students will start with a sample of cesium-137 (^{137}Cs) that will decay into an excited state ^{137m}Ba which has a half-life of only a few minutes. When the ^{137m}Ba decays into the ground state of ^{137}Ba, it emits a gamma ray where the energy of the gamma ray is recorded by a multichannel analyzer (MCA). The exponential decay of radioactive nuclides is parameterized by parameter called the *half-life* ($t_{1/2}$).

I. BACKGROUND

The process of radioactive decay is purely stochastic, that is, it is completely random. Suppose you have a sample containing N_o radioactive nuclei. A tempting question to ask is, "How long will it take for a specific nucleus to decay?" Unfortunately, it is impossible to determine when a particular nucleus is going to spontaneously decay. A more reasonable question to ask might relate to the activity of the sample. In other words, "How many nuclei in the sample will radioactively decay in one second? ... or in one minute?" for example. Probing radioactive decays from this perspective provides a physical parameter, namely the *half-life* by which we can characterize individual radioisotopes.

II. THEORY

In this section we describe a brief introduction to the theory of radioactive decay. Radioactive decay is entirely due to the *weak* force. In the experiment you will be performing, you will observe the following two-step nuclear-chemical reaction:

$$^{137}Cs \ \rightarrow \ ^{137m}Ba + e^- + \bar{\nu}_e \tag{1a}$$

$$^{137m}Ba \ \rightarrow \ ^{137}Ba + \gamma \tag{1b}$$
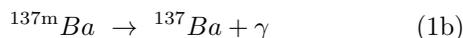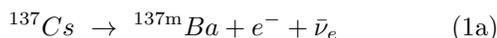
It is the *second* of these two equations (Eq. 1b) that you will want to focus your attention. Let's assume we have a sample of N_o radioactive nuclides of a particular isotope (e.g., ^{137m}Ba). The number of nuclei that decay ($N - N_o$) in a time dt can be written as:

$$dN = -\lambda N \, dt \tag{2}$$

where $dN = N_f - N_i$ is an intrinsically negative number (final - initial), N is the instantaneous number of radioactive nuclides present, and λ is the decay rate for a *single* radionuclide. Separating variables (i.e., gathering the "N" terms to one side of the equation) and integrating both sides of this equation, we find:

$$\int_{N_o}^{N} \frac{dN}{N} \ = \ \lambda \int_0^t dt$$

which leads to the following equation:

$$N(t) \ = \ N_o \, e^{-\lambda t} \tag{3}$$

the most commonly used expression for describing the *number of radioactive isotopes in a sample* as a function of time. The only problem is, we don't know how many radioactive isotopes we have in a sample. What we measure is the number of radioactive decays. This can be easily calculated by taking a derivative of the above equation (Eq. 3):

$$\frac{dN}{dt} \ = \ (-\lambda \, N_o) \, e^{-\lambda t}$$

where λN_o is called the *activity* of the radioactive sample (i.e., counts/sec). Since dN/dt is an intrinsically negative number in this equation, this equation is sometimes written as:

$$\left| \frac{dN}{dt} \right| \ = \ (\lambda \, N_o) e^{-\lambda t} \tag{4}$$

which is the equation you will be using to describe the data you collect. The initial activity of your sample (λN_o) is measured in counts/sec. Another unit that is commonly used is the curie (Ci) which is 3.7×10^{10} decays/sec. If you are in a room containing an *unshielded* 1 Ci source, immediately leave the room. Most radioactive sources that we will be using in the modern physics lab are 10-100 μCi (micro-curies).

III. THE EXPERIMENT

In this experiment, you are going to measure the number of gamma-ray (γ) per second entering your detector and record this rate as a function of time. The gamma rays are from the reaction described in Eq. 1b. An example of this decay process is shown in Fig. 1 where the *counts/sec* $|dN/dt|$ is plotted as a function of time.

As described in Fig. 1, one must add a *constant* term to Eq. 3 in order to obtain a good fit. The constant term takes into account the background radiation in the room that can *creep* into the experiment. In other words, after the radioactive source has completely decayed, one would still measure a "flat" number of counts/sec in the detector due to the background sources.
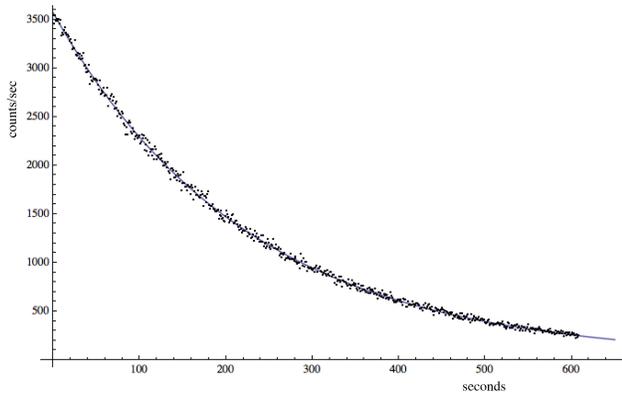
FIG. 1. A typical exponential decay describes the observed counts/sec as seen in this radioactive decay experiment. The best fit to this data $dN/dt = \lambda N_o e^{-\lambda t} + \text{const.}$ is shown overlaid on the data. The initial activity in this experiment (λN_o) is seen to be just over 3500 counts/sec. The dwell time (the time interval of each bin) is one second.

IV. THE EQUIPMENT

The equipment and procedure is described in section of the manuals found on my `physicsx` website. The manuals are too long to read completely, so, let me focus your attention on the relevant material. First of all, a description of the detector can be found in the leaflet called, "Energy Calibration of the PMT." The PMT is short for photomultiplier tube.

The gamma ray from the barium-137m decay will produce light in the sodium-iodide crystal (NaI) attached to the end of the PMT. The light incident on the photocathode surface of the PMT produces photoelectrons that are multiplied as they avalanche down the dynode chain. The gain for a single photoelectron is typically $500,000\times$ to $1,000,000\times$. The number of photoelectrons produced (pe's) at the beginning of the avalanche is proportional to the energy of the γ ray captured in the NaI crystal. In other words, the size of the pulse coming out of the PMT is proportional to the energy of the γ ray emitted from the barium-137m.

The signal coming out of the PMT is sent to the multichannel analyzer (MCA). The MCA unit is described in Experiment 8 in the writeup titled, "Nuclear Spectroscopy" as found on my `physicsx` website. Experiment 9 describes counting statistics which is important for calculating the error-bars in your experiment. You will have another experiment later on that investigates counting statistics. The important piece of information to glean from the information described in Experiment 9 is that the error bars are equal to $\sqrt{dN/dt}$, or the square-root of the number of counts in that bin. What that means is that your error bars will start out large in the early part of your data collection, and get progressively smaller as the number of counts is observed to reduce over time.

V. PROCEDURE

When the MCA is operating in the correct mode, it will plateau the PMT (i.e., find the best operating voltage), typically 500-700 volts for these PMTs. I would follow the suggested experimental procedure as described in Experiment 8. However, I would select a dwell time of 1 second instead of 4 seconds.

The procedure also directs you to take a background measurement. I find that unnecessary if you use a *constant* term in the fitting function as described in Fig. 1.

It's important to select *lower-* and *upper-limits* for you energy cutoffs, so, you don't include spurious sources as part of your data collection. You only want to include counts that fall within that energy *window*.

The goal of this experiment is measure the half-life of barium-137m ($t_{1/2}$). One of the parameters (λ) in your fitting function is related to the half-life. There is a simple relationship between λ and the half-life ($t_{1/2}$).

$$\lambda = \frac{\ell n 2}{t_{1/2}} = \frac{0.693}{t_{1/2}} \tag{5}$$

You might want to consider rewriting your fitting function using the above value of lambda (Eq. 5) so you can obtain the half-life directly from your fitting function and your χ^2 analysis.

VI. IMPORTANT CONSIDERATIONS

- You should be able to obtain the correct lifetime to within ~1%.

- Have your instructor or lab assistant help you prepare your first radioactive source.

- There are many pieces to this experiment; however, it should not be beyond your abilities to understand what is happening every step of the way.

- The fitting function is non-linear, so, you must use a computer program that does multidimensional minimization of the χ^2. You have 3 parameters in your fitting function, so, it's a multi-dimensional fit in χ^2 space.