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TOPIC	

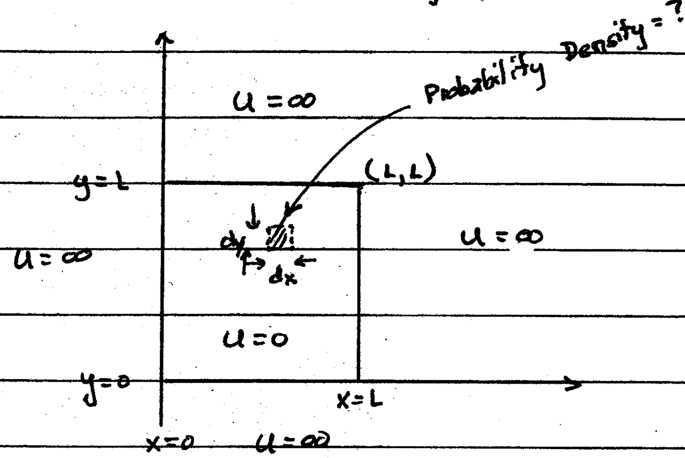
Two-Dimensional Potential Well

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)+U(x,y)\,\psi = E\,\psi \qquad \psi=\psi(x,y)$$

Assume the solutions are separable: $\psi(x,y)=\psi_x(x)\,\psi_y(y)$

B.C.'s $\Rightarrow$ $\psi(0,y)=0$ & $\psi(L,y)=0$ for all "y"

$\Rightarrow$ $\psi(x,0)=0$ & $\psi(x,L)=0$ for all "x"



If $\psi(x,y)$ is separable, then

$$\frac{-\hbar^2}{2m}\left(\psi_y\frac{\partial^2\psi_x}{\partial x^2}+\psi_x\frac{\partial^2\psi_y}{\partial y^2}\right)+0\,\psi_x\psi_y=(E_x+E_y)\,\psi_x\psi_y$$

$\div$ by $\psi(x,y)$ $$\frac{-\hbar^2}{2m}\left(\frac{1}{\psi_x}\frac{\partial^2\psi_x}{\partial x^2}+\frac{1}{\psi_y}\frac{\partial^2\psi_y}{\partial y^2}\right)=E_x+E_y$$

Two Equations $$-\frac{\hbar^2}{2m}\frac{1}{\psi_x}\frac{\partial^2\psi_x}{\partial x^2}=E_x \quad \text{and} \quad -\frac{\hbar^2}{2m}\frac{1}{\psi_y}\frac{\partial^2\psi_y}{\partial y^2}=E_y$$

(1) $$\frac{\partial^2\psi_x}{\partial x^2}+\frac{2mE_x}{\hbar^2}\psi_x=0$$ (2) $$\frac{\partial^2\psi_y}{\partial y^2}+\frac{2mE_y}{\hbar^2}\psi_y=0$$

$$\psi_x(x)=A\sin k_x x+B\cos k_x x \qquad \psi_y(y)=C\sin k_y y+D\cos k_y y$$

Applying the B.C.'s: $$\psi(x,y)=\psi_x(x)\,\psi_y(y)=A'\sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right)$$

Normalization: $$1=\int_0^L\int_0^L\psi^*(x,y)\,\psi(x,y)\,dx\,dy=(A')^2\int_0^L dy\,\sin^2\left(\frac{n\pi y}{L}\right)\int_0^L dx\,\sin^2\left(\frac{n\pi x}{L}\right)$$

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Normalization cont'd

$$1 = (A')^2 \frac{L}{2} \frac{L}{2} \qquad A' = \frac{2}{L}$$

$$\boxed{\psi_{n_x,n_y}(x,y) = A' \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{n\pi y}{L}\right) = \frac{2}{L} \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{n\pi y}{L}\right)}$$

Energy Eigenvalues: $\boxed{E = E_x + E_y = \frac{\hbar^2 \pi^2}{2mL^2}\left(n_x^2 + n_y^2\right)}$ $\quad n_x = 1, 2, 3, \ldots$ $\quad n_y = 1, 2, 3, \ldots$

The wavefunction $\psi_{n_x, n_y}(x,y)$ has $\underline{\underline{2}}$ quantum numbers.

Some of the energy states are degenerate.

For example: $E_{1,1} = \frac{\hbar^2\pi^2}{2mL^2}(1+1) = \frac{\hbar^2\pi^2}{mL^2} = 2E_0$

$E_{1,1} = 2E_0$

$E_{2,1} = E_{1,2} = 5E_0$ (degenerate) ← Two different quantum states with the same energy.

$E_{2,2} = 8E_0$

$E_{3,1} = E_{1,3} = 10E_0$ (degenerate)

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