Simple Harmonic Oscillator

$U(x) = \frac{1}{2}kx^2$

$k = m\omega^2$

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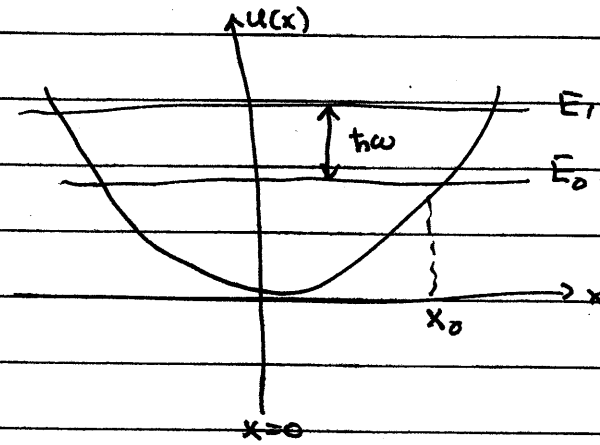

Schrödinger Equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2x^2\psi = E\psi$$

$$\xi = \sqrt{\frac{m\omega}{\hbar}}\,x = \beta x \qquad \beta = \sqrt{\frac{m\omega}{\hbar}}$$

$$\frac{d^2\psi}{d\xi^2} = (\xi^2 - K)\psi \quad \text{where} \quad K = \frac{2E}{\hbar\omega}$$

Physically acceptable solutions require $K = 2n+1$

$$\boxed{\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}H_n(\xi)\,e^{-\xi^2/2}} \qquad n = 0, 1, 2, 3, \ldots$$

$H_0 = 1$

$H_1 = 2\xi = 2\beta x$

$H_2 = 4\xi^2 - 2 = 4\beta^2x^2 - 2$

$H_3 = 8\xi^3 - 12\xi = 8\beta^3x^3 - 12\beta x$

$\vdots$

$$\boxed{E_n = \left(n + \tfrac{1}{2}\right)\hbar\omega_0 \quad n = 0, 1, 2, \ldots}$$

$\omega_0 = \frac{k}{m}$ $\qquad$ $x_0 = \sqrt{\frac{2E}{k}}$

Turning Point

$\frac{1}{2}kx_0^2 = E$

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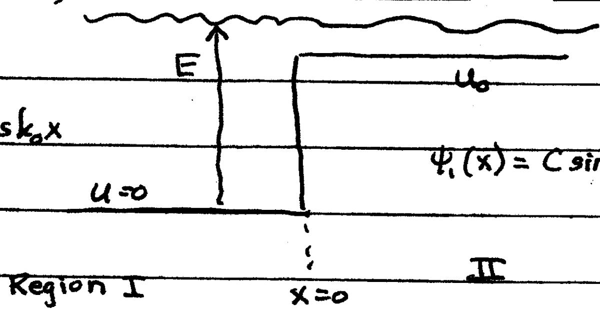
Potential Energy Step, $E > U_0$



$$\psi_0 = A \sin k_0 x + B \cos k_0 x$$

$$\psi_1(x) = C \sin k_1 x + D \cos k_1 x$$

$$k_0 = \sqrt{\frac{2mE}{\hbar^2}} \qquad k_1 = \sqrt{\frac{2m}{\hbar^2}(E - U_0)}$$

$$\sin k_0 x = \frac{e^{ik_0 x} - e^{-ik_0 x}}{2i} \qquad \cos k_0 x = \frac{e^{ik_0 x} + e^{-ik_0 x}}{2}$$

⋮

Eq. 55 (a) & (b)

$$\boxed{\psi_0(x) = A' e^{ik_0 x} + B' e^{-ik_0 x} \quad x < 0} \quad \text{\&} \quad \boxed{\psi_1(x) = C' e^{ik_1 x} + D' e^{-ik_1 x} \quad x > 0}$$

$|A'|^2$ = intensity of the wave moving left → right
$|B'|^2$ = intensity of the wave moving right → left. } In region I
$|C'|^2$ = intensity of the wave moving left → right
$|D'|^2$ = 0 (usually), because the initial flux of particles is from the left.

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Problem 28

$x=0$ $\quad A' + B' = C'$

$A' = C' - B'$

$$\cancel{A' i k_0 - B' i k_0 = i k_1 C'}$$
$$\cancel{(C' - B') i k_0 - i k_0 B' = i k_1 C'}$$
$$\cancel{i k_0 C' - i k_0 (2B') = i k_1 C'}$$

$$i k_0 A' - i k_0 B' = i k_1 C'$$
$$A' + B' = C'$$

$$i k_0 A' - i k_0 B' = i k_1 A' + i k_1 B'$$
$$A'(i k_0 - i k_1) = B'(i k_0 + i k_1)$$
$$B' = A'\left(\frac{i k_0 - i k_1}{i k_0 + i k_1}\right) = A'\left(\frac{k_0 - k_1}{k_0 + k_1}\right)$$

Reflection Probability $= \left|\frac{B'}{A'}\right|^2 = \left(\frac{k_0 - k_1}{k_0 + k_1}\right)^2 = \left(\frac{1 - \frac{k_1}{k_0}}{1 + \frac{k_1}{k_0}}\right)^2$

$$\frac{k_1}{k_0} = \frac{\sqrt{\frac{2m}{\hbar^2}(E - U_0)}}{\sqrt{\frac{2M}{\hbar^2} E}} = \sqrt{\frac{E - U_0}{E}} = \sqrt{1 - \frac{U_0}{E}}$$