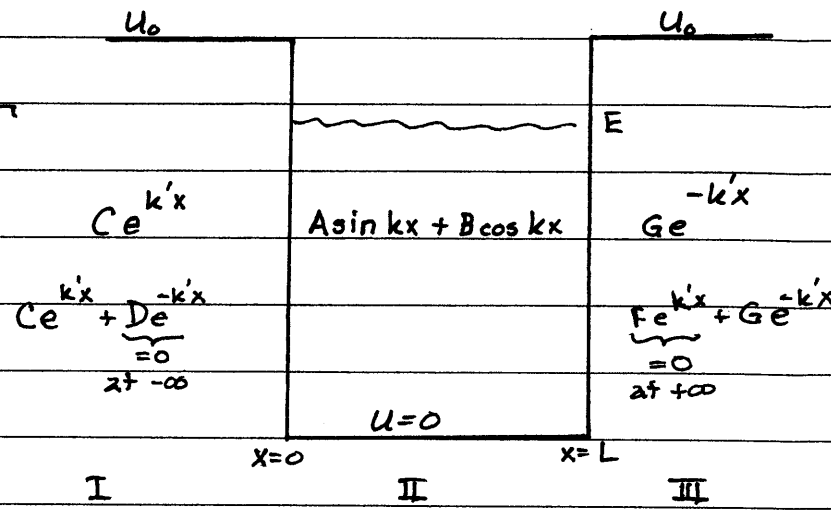
PREPARED BY

DATE 1

Finite Potential Well

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$k' = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} \qquad \beta = \sqrt{\frac{2mU_0}{\hbar^2}}$$



Boundary Conditions.

$\underline{x=0}$

(1) $C = B$

(2) $k'C = Ak$

$\underline{x=L}$

(3) $A\sin kL + B\cos kL = Ge^{-k'L}$

(4) $Ak\cos kL - Bk\sin kL = -k'Ge^{-k'L}$

(2) $\dfrac{k'}{k} = r = \dfrac{A}{C}$ $\quad \boxed{A = rC} \qquad \dfrac{k'}{k} = \sqrt{\dfrac{U_0 - E}{E}} = \sqrt{\dfrac{U_0}{E} - 1}$

(3) $rC\sin kL + C\cos kL = Ge^{-k'L}$

(4) $rC\cos kL - C\sin kL = -rGe^{-k'L}$

(4) ÷ (3) $\dfrac{rC\cos kL - C\sin kL}{rC\sin kL + C\cos kL} = \dfrac{-rGe^{-k'L}}{Ge^{-k'L}} = -r$

$$\frac{r\cos kL - \sin kL}{r\sin kL + \cos kL} = -r \Rightarrow \boxed{\frac{r - \tan kL}{r\tan kL + 1} = -r}$$

$$r - \tan kL = -r^2\tan kL - r \Rightarrow \boxed{2r = (1 - r^2)\tan kL}$$

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DATE (2)

Finite Potential Well

$$2 \cot kL = \frac{(1-r^2)}{r} = \frac{(1-k'^2/k^2)}{k'/k} = \frac{(k^2-k'^2)}{k\,k'}$$

$$2k \cot kL = \frac{k^2-k'^2}{k'} \qquad 2(kL)\cot(kL) = \frac{(kL)^2-(k'L)^2}{k'L}$$

Define: $\boxed{Z = kL \qquad Z_o = \beta L \qquad k'^2 = \beta^2 - k^2}$

$$2z \cot z = \frac{z^2-(\beta^2-k^2)L^2}{\sqrt{\beta^2-k^2}\,L} = \frac{z^2 - z_o^2 + z^2}{\sqrt{z_o^2-z^2}} = \frac{2z^2-z_o^2}{z\sqrt{z_o^2/z^2-1}}$$

$$\boxed{2 \cot z = \frac{\left(2 - z_o^2/z^2\right)}{\sqrt{z_o^2/z^2 - 1}}}$$

Where $z_o = \beta L = \sqrt{\dfrac{2mU_o}{\hbar^2}}\,L$

Solve this for an electron $L = 0.100$ nm

$U_o = 400$ eV

$z = \{2.62443,\ 5.21678,\ 7.72156,\ 9.93333\}$ from Mathematica

only 4 roots!

$$k = \frac{z}{L} = \{26.2443,\ 52.1678,\ 77.2156,\ 99.3333\}\ \text{nm}^{-1}$$

E = energy eigenvalues $\Rightarrow$ $E_i = \dfrac{\hbar^2 k_i^2}{2m_e} = \dfrac{(\hbar c)^2 k_i^2}{2m_e c^2}$

$$\boxed{E_i = \{26.15,\ 103.3,\ 226.4,\ 374.7\}\ \text{eV}}$$

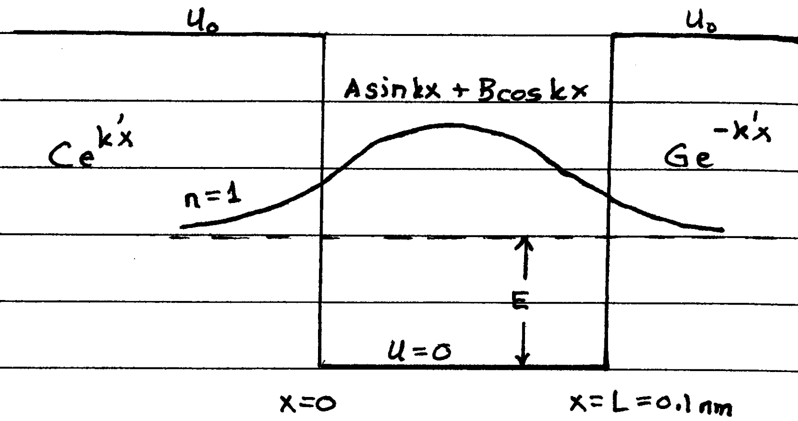
Finite Potential Wave Function

Recall $\Rightarrow$ $B = C$

$k'C = Ak$

$$A = \frac{k'}{k} C = rC$$

$$= \sqrt{\frac{U_0}{E} - 1}\, C$$



$$\psi(x) \Rightarrow \begin{cases} Ce^{+k'x} & x \le 0 \\ rC\sin kx + C\cos kx & 0 \le x \le L \\ \dfrac{C(r\sin kL + \cos kL)}{e^{-k'L}}\, e^{-k'x} & x \ge L \end{cases}$$

where $k' = rk$

We know k_i's, so, we also know k_i''s.

Use the normalization of the wave function to obtain C. ($n=1$)

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$$

$U_0 = 400$ eV
$E_1 = 26.15$ eV

$k \to k_1 = 26.24\ nm^{-1}$
$k' \to k_1' = 99.22\ nm^{-1}$
$E \to E_1 = 26.15$ eV

$$1 = C^2 \int_{-\infty}^{0} e^{2k'x} + C^2 \int_0^L (r\sin kx + \cos kx)^2 dx + \frac{C^2 (r\sin kL + \cos kL)^2}{e^{-2k'L}} \int_L^{\infty} e^{-2k'x} dx$$

$$\boxed{C = 1.04325}$$

Using this value of "C," we can construct the piecewise-continuous wave function from $-\infty$ to ∞.

for the $n=1$ wave function