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Classical Theory

$$N(\lambda)\,d\lambda = \frac{8\pi V}{\lambda^4}\,d\lambda$$

$E \Rightarrow$ "continuous"

$$N(E) = \frac{N}{kT}\,e^{-E/kT}$$

← normalized pdf

$$\boxed{E_{AV} = \frac{1}{N}\int_0^\infty E\,N(E)\,dE = kT}$$

$$u(\lambda)\,d\lambda = \frac{N(\lambda)\,d\lambda}{V}\,(kT)$$

$$u(\lambda)\,d\lambda = \left(\frac{8\pi kT}{\lambda^4}\right)d\lambda$$

$$I(\lambda)\,d\lambda = \frac{c}{4}\,u(\lambda)\,d\lambda$$

$$I(\lambda) = \frac{2\pi c\,kT}{\lambda^4}$$

$$\boxed{I(\lambda)\,d\lambda = \frac{2\pi c\,kT}{\lambda^4}\,d\lambda}$$

SAME ⟺

Quantum Theory

$$N(\lambda)\,d\lambda = \frac{8\pi V}{\lambda^4}\,d\lambda$$

$E_n = n\varepsilon \quad (n = 1, 2, 3, \ldots) \qquad \varepsilon = hf$

"discrete"

$$N_n(E_n) = N\left(1 - e^{-\varepsilon/kT}\right)e^{-E_n/kT}$$

$$N_n(\varepsilon) = N\left(1 - e^{-\varepsilon/kT}\right)e^{-n\varepsilon/kT}$$

← normalized pdf.

$$E_{AV} = \frac{1}{N}\sum_{n=0}^{\infty} N_n E_n$$

$$E_{AV} = \left(1 - e^{-\varepsilon/kT}\right)\sum_{n=0}^{\infty}(n\varepsilon)\,e^{-n\varepsilon/kT}$$

$$\boxed{E_{AV} = \frac{\varepsilon}{e^{\varepsilon/kT} - 1} = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1}}$$

$$u(\lambda)\,d\lambda = \frac{N(\lambda)\,d\lambda}{V}\left(\frac{hc/\lambda}{e^{hc/\lambda kT} - 1}\right)$$

$$u(\lambda)\,d\lambda = \frac{8\pi}{\lambda^4}\left(\frac{hc/\lambda}{e^{hc/\lambda kT} - 1}\right)$$

$$I(\lambda)\,d\lambda = \frac{c}{4}\,u(\lambda)\,d\lambda$$

$$\boxed{I(\lambda)\,d\lambda = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda kT} - 1\right)}\,d\lambda}$$

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TOPIC

Power / Area Blackbody Radiation

$$I = \int_0^\infty I(\lambda)\, d\lambda = 2\pi h c^2 \int_0^\infty \frac{d\lambda}{\lambda^5 \left(e^{hc/\lambda kT} - 1\right)}$$

$$I = \left(\frac{2\pi^5 k^4}{15 c^2 h^3}\right) T^4 = \sigma T^4 \quad \left[\frac{\text{Power}}{\text{Area}}\right]$$

$$\boxed{\sigma = 5.67 \times 10^{-8} \ \frac{W}{m^2 K^4}}$$

Wien's Displacement Law

$$\lambda_{max} T = 2.8978 \times 10^{-3} \ m \cdot K$$

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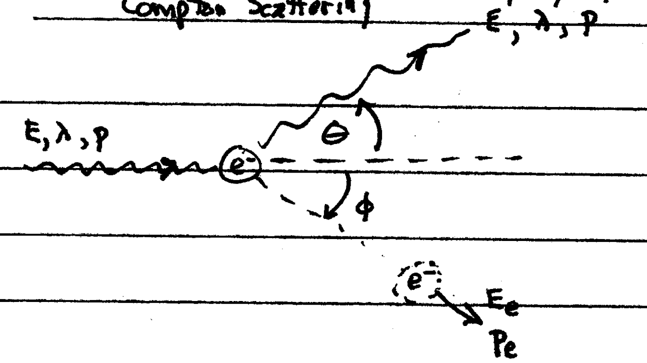
TOPIC	

Compton Scattering



① $E_i = E_f \qquad E_\gamma + m_e c^2 = E'_\gamma + E_e \qquad E + m_e c^2 = E' + E_e$

② $p_{x,i} = p_{x,f} \qquad p = p'\cos\theta + p_e\cos\phi \quad \rightarrow \quad \cancel{p_e\cos\phi \neq p'\cos\theta}$

$$p_e\cos\phi = p - p'\cos\theta$$

③ $p_{y,i} = p_{y,f} \qquad 0 = p'\sin\theta - p_e\sin\phi \qquad p_e\sin\phi = p'\sin\theta$

$$p_e^2\cos^2\phi = p^2 - 2pp'\cos\theta + p'^2\cos^2\theta$$
$$p_e\sin^2\phi = \qquad p'^2\sin^2\theta$$
$$p_e^2 = p^2 - 2pp'\cos\theta + p'^2$$

① $(E + m_e c^2 - E')^2 = (E_e)^2 = c^2 p_e^2 + m_e^2 c^4$

$$(E + m_e c^2 - E')^2 = c^2(p^2 - 2pp'\cos\theta + p'^2) + m_e^2 c^4$$
$$\left[(E - E') + m_e c^2\right]^2 = p^2c^2 + p'^2c^2 - 2pp'c^2\cos\theta + m_e^2 c^4$$
$$E^2 + E'^2 - 2EE' + 2m_e c^2(E - E') + \underline{m_e^2 c^4} = p^2c^2 + p'^2c^2 - 2pp'c^2\cos\theta + \underline{m_e^2 c^4}$$

$$\underbrace{(E^2 - p^2c^2)}_{=0} + \underbrace{(E'^2 - p'^2c^4)}_{=0} - 2EE' + 2m_e c^2(E - E') = -2pp'c^2\cos\theta$$

$$2m_e c^2(E - E') = 2EE'(1 - \cos\theta) \Rightarrow (E - E') = \frac{EE'}{m_e c^2}(1 - \cos\theta)$$

$$hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{hc}{\lambda\lambda'} \; \frac{hc(1-\cos\theta)}{m_e c^2} \Rightarrow \boxed{\lambda' - \lambda = \left(\frac{h}{m_e c}\right)(1 - \cos\theta)}$$

$\boxed{\frac{h}{m_e c} = 0.002426 \text{ nm}}$ ⇐ Compton Wavelength

Compton Scattering

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Compton Scattering

The Direction of the Electron's Motion $\rightarrow \phi$

$$\boxed{\tan\phi = \frac{E'\sin\theta}{E - E'\cos\theta}}$$