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1.9 A container holds N molecules of N_2 gas, $T = 280\,K$

Find the number of molecules with KE's between $0.0300\,eV \rightarrow 0.0312\,eV$

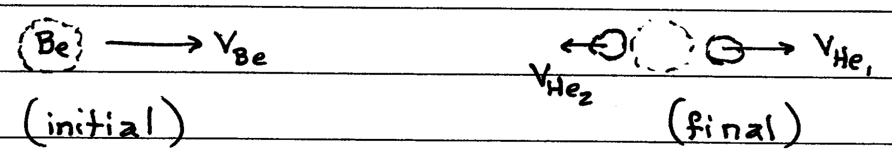
$$kT = 8.617 \times 10^{-5}\,eV/K\,(280\,K) = 0.0241\,eV \qquad dE = 0.0012\,eV$$

$$N(E)\,dE = \frac{2N}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}\left(E_{mid\text{-}point}\right)^{1/2} e^{-\frac{E_{mid\text{-}point}}{kT}}\,dE$$

$$N(E)\,dE = N\,\frac{2}{\sqrt{\pi}}\frac{1}{(.0241\,eV)^{3/2}}(0.0306\,eV)^{1/2}\,e^{-\frac{(0.0306\,eV)}{.0241\,eV}}\,(0.0012\,eV)$$

$$\boxed{N(E)\,dE = 0.0178} \times N$$

1.13 Be atom in problem 4 is moving with $KE = 40\,keV$ in the $+x$ direction



a.) Use conservation of momentum and energy:

$$KE = \frac{P_{Be}^2}{2m_{Be}} \qquad P_{Be} = \sqrt{2m_{Be}\,KE} = \sqrt{2(8u)\,931.494\,MeV/c^2\;40\times10^{-3}\,MeV}$$

$$\boxed{P_{Be} = 24.41\,MeV/c} = P_{initial}$$

Cons. of Momentum: $P_{initial} = P_{final}$ $\qquad \underbrace{P_{initial}}_{P_{Be}} = P_{He_1} + P_{He_2}$

Cons. of Energy: $\underbrace{Q}_{92.2\,keV} + \frac{P_{Be}^2}{2m_{Be}} = \frac{1}{2m_{He}}\left(P_{He_1}^2 + P_{He_2}^2\right)$

$$2m_{He}Q + \frac{m_{He}}{m_{Be}}P_{Be}^2 = P_{He_1}^2 + P_{He_2}^2$$

$P_{He_2} = P_{Be} - P_{He_1}$ from Cons. of Momentum

$$2m_{He}Q + \frac{1}{2}P_{Be}^2 = P_{He_1}^2 + (P_{Be} - P_{He_1})^2$$

Solve this Quadratic Equation

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1.13 Cont'd Using Mathematica $\longrightarrow$ $\boxed{P_{He_1} = 30.743\ \text{MeV/c}}$

$$V_{He_1} = \frac{P_{He_1}}{m_{He}} = \frac{30.743\ \text{MeV/c}}{4u\left(931.494\ \frac{\text{MeV/c}^2}{u}\right)} = 8.25\times10^{-3}\ c$$

$$\boxed{V_{He_1} = 2.474\times10^{6}\ \frac{m}{s}}$$

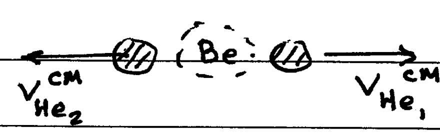
$$P_{He_2} = m_{He} V_{He_2} = P_{Be} - P_{He_1} \quad \text{from cons. of momentum}$$

$$V_{He_2} = \frac{P_{Be} - P_{He_1}}{m_{He}} = \frac{24.41\ \text{MeV/c} - 30.74\ \text{MeV}}{4u\left(931.494\ \frac{\text{MeV/c}^2}{u}\right)} = -1.70\times10^{-3}\ c$$

$\boxed{V_{He_2} = -5.09\times10^{+5}\ m/s}$ It's moving in the $-x$ direction.

b.) Using the S and S' reference frames. S' = Be CM frame

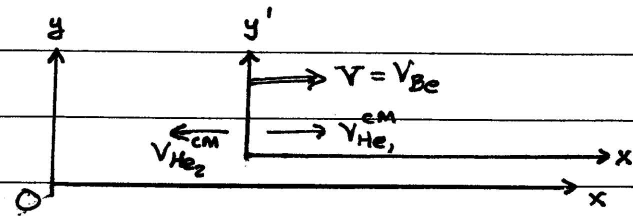
S' frame $\rightarrow$ Be is at rest.



Recall that $Q = 92.2\ \text{keV} = \dfrac{P_{He_2}^2}{2m_{He}} + \dfrac{P_{He_1}^2}{2m_{He}} = 2\left(\dfrac{1}{2} m_{He}\left(V_{He}^{cm}\right)^2\right)$

$$Q = m_{He} V_{He,cm}^2 \qquad V_{He,cm} = \sqrt{\frac{Q}{m_{He}}} = \sqrt{\frac{92.2\times10^{-3}\ \text{MeV}}{4u\left(931.494\ \frac{\text{MeV/c}^2}{u}\right)}}$$

$$\boxed{V_{He}^{CM} = 1.4913\times10^{6}\ m/s}$$



$$K_{Be} = \frac{1}{2} m_{Be} V_{Be}^2$$

$$V_{Be} = \sqrt{\frac{2K_{Be}}{m_{Be}}}$$

$$V_{Be} = \sqrt{\frac{2\left(40\times10^{-3}\ \text{MeV}\right)}{8u\left(931.494\ \frac{\text{MeV}}{c^2/u}\right)}} = 9.823\times10^{5}\ m/s$$

$$V_{He_1}^{Lab} = V_{Be} + V_{He_1}^{cM} = \left(9.823\times10^{5} + 1.4913\times10^{6}\right) m/s \qquad \boxed{V_{He_1}^{Lab} = 2.47\times10^{6}\ m/s}$$

$$V_{He_2}^{Lab} = V_{Be} + V_{He_2}^{cM} = \left(9.823\times10^{5} - 1.4913\times10^{6}\right) m/s \qquad \boxed{V_{He_2}^{Lab} = -5.09\times10^{5}\ m/s}$$

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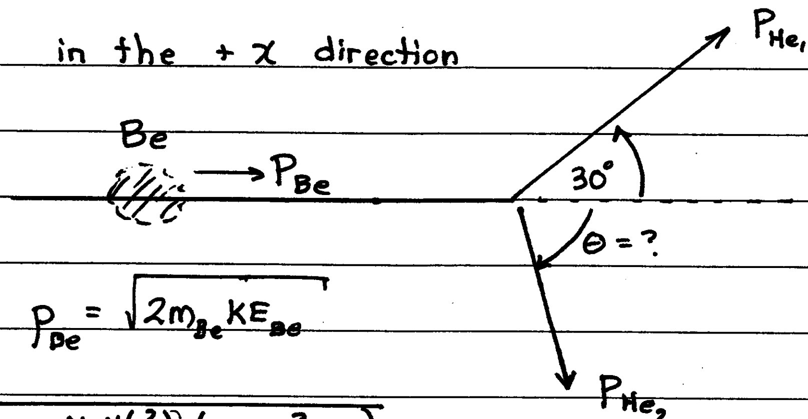
1.14 Beryllium atom from problem 4

$KE_{Be} = 60 \text{ keV}$ in the $+x$ direction



$$KE_{Be} = \frac{P_{Be}^2}{2m_{Be}} \qquad P_{Be} = \sqrt{2m_{Be} KE_{Be}}$$

$$P_{Be} = \sqrt{2(8u)\left(931.494 \tfrac{MeV/c^2}{u}\right)\left(60\times10^{-3} \text{ MeV}\right)}$$

$$\boxed{P_{Be} = 29.90 \text{ MeV/c}}$$

a.) Use conservation of momentum and energy. $Q = 92.2 \text{ keV}$

Cons. of Momentum:

$$x: \quad P_{Be} = P_{He_1}\cos 30^\circ + P_{He_2}\cos\theta$$
$$y: \quad 0 = P_{He_1}\sin 30^\circ - P_{He_2}\sin\theta$$

Square both of these equations (x: and y:) and add them.

$$x: \quad P_{He_2}\cos\theta = -P_{He_1}\cos 30^\circ + P_{Be}$$
$$y: \quad P_{He_2}\sin\theta = +P_{He_1}\sin 30^\circ$$

$$(x:)^2 \quad P_{He_2}^2\cos^2\theta = P_{He_1}^2\cos^2 30^\circ - 2P_{Be}P_{He_1}\cos 30^\circ + P_{Be}^2$$
$$(y:)^2 \quad P_{He_2}^2\sin^2\theta = P_{He_1}^2\sin^2 30^\circ$$

Add $\longrightarrow$ $$\boxed{P_{He_2}^2 = P_{He_1}^2 - 2P_{Be}P_{He_1}\cos 30^\circ + P_{Be}^2}$$ ← save This

Cons. of Energy

$$\frac{P_{Be}^2}{2m_{Be}} + Q = \frac{1}{2m_{He}}\left(P_{He_1}^2 + P_{He_2}^2\right)$$

$$\frac{m_{He}}{m_{Be}}P_{Be}^2 + 2m_{He}Q = P_{He_1}^2 + P_{He_2}^2$$

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1.14 Cont'd

$$\left(\frac{m_{He}}{m_{Be}}\right)P_{Be}^2 + 2m_{He}Q = P_{He_1}^2 + P_{He_1}^2 - 2P_{Be}P_{He_1}\cos 30^\circ + P_{Be}^2$$

$\frac{1}{2}$

$$2P_{He_1}^2 - (2p_{Be}\cos 30^\circ)P_{He_1} + \left(\frac{1}{2}P_{Be}^2 - 2m_{He}Q\right) = 0$$

This is a quadratic equation in P_{He_1}. Using Mathematica:

$$P_{He_1} = 29.91 \text{ MeV/c} \qquad \boxed{V_{He_1} = \frac{P_{He_1}}{m_{He}} = 2.407\times10^6 \text{ m/s}}$$

$$\tan\theta = \frac{(P_{He_2})_y}{(P_{He_2})_x} = \frac{P_{He_1}\sin 30^\circ}{P_{Be} - P_{He_1}\cos 30^\circ} = 3.73676$$

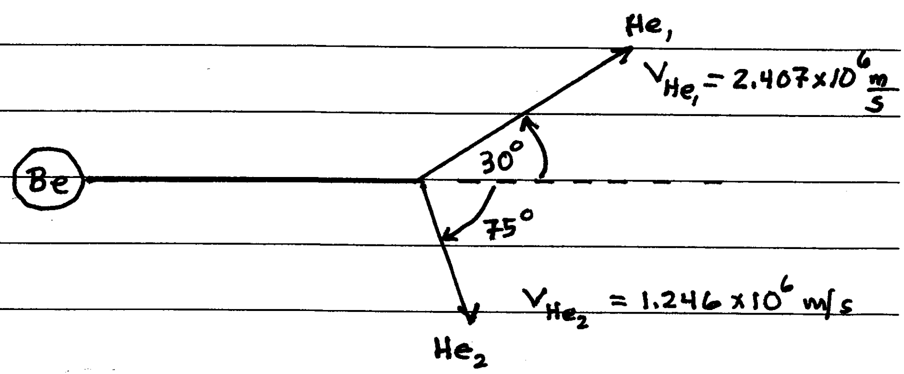
$$\boxed{\theta = \text{Tan}^{-1}(3.737) = 75.02^\circ}$$

y: $P_{He_2}\sin\theta = P_{He_1}\sin 30^\circ \qquad P_{He_2} = \frac{P_{He_1}\sin 30^\circ}{\sin(75.02^\circ)}$

$$\boxed{P_{He_2} = 15.48 \text{ MeV/c}} \qquad V_{He_2} = \frac{P_{He_2}}{m_{He}} = \frac{15.48 \text{ MeV/c}}{4u\left(931.494 \frac{\text{MeV/c}^2}{u}\right)}$$

$$\boxed{V_{He_2} = 1.246\times10^6 \text{ m/s}}$$

So, from conservation of momentum and energy, we have the following picture:



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1.14 Cont'd

b.) Use velocity vectors between the laboratory frame and the center-of-mass frame.

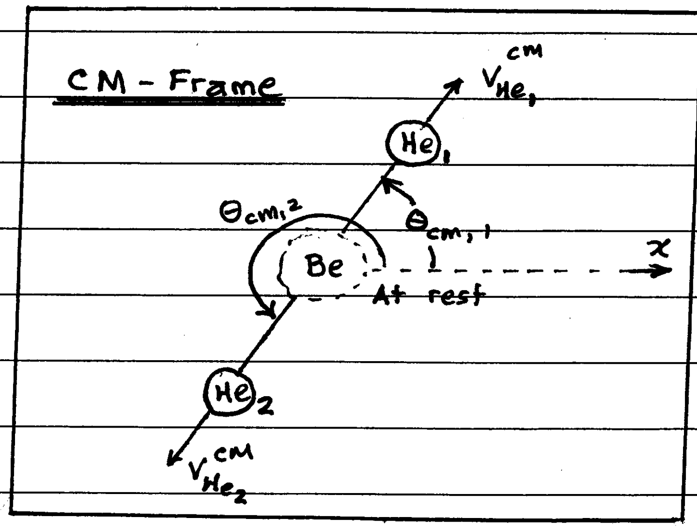
In the laboratory frame: $V_{Be} = \frac{P_{Be}}{M_{Be}} = 1.203 \times 10^{6}$ m/s

This is also V_{cm}. $\boxed{V_{cm} = 1.203 \times 10^{6} \text{ m/s}}$

The velocity of the He atoms in the CM system is:

$$\frac{1}{2} m_{He} \left(V_{He}^{CM}\right)^2 = \frac{Q}{2} \qquad V_{He}^{CM} = \sqrt{\frac{Q}{m_{He}}} = \sqrt{\frac{92.2 \times 10^{-3} \text{ MeV}}{4u\left(931.494 \frac{\text{MeV}/c^2}{u}\right)}}$$

$$\boxed{V_{He}^{CM} = 1.4913 \times 10^{6} \text{ m/s}}$$



In the LAB FRAME: for the He$_1$ atom,

$$V_{x_1} = V_{x,1}^{CM} + V_{Be}$$
$$V_{y_1} = V_{y,1}^{CM}$$

$$V_{x_1} = V_{He_1}^{CM} \cos\theta_{CM} + V_{Be}$$
$$V_{y_1} = V_{He_1}^{CM} \sin\theta_{CM}$$

Where $\boxed{\tan 30^\circ = \frac{V_{y_1}}{V_{x_1}} = \frac{V_{He_1}^{CM} \sin\theta_{CM}}{V_{He_1}^{CM} \cos\theta_{CM} + V_{Be}}}$

The only unknown is θ_{CM}. However, this is a transcendental equation "in θ_{CM}." Using Mathematica we find $\boxed{\theta_{CM} = 53.79^\circ}$

for He$_1$

For He$_2$ $\theta_{CM}(He_2) = \theta_{CM} + 180^\circ$

$$\boxed{\theta_{CM}(He_2) = 233.79^\circ} = \theta_{CM,2}$$

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1.14 Cont'd

$$V_{x,1} = V_{He}^{CM} \cos\theta_{cm,1} + V_{Be} = 2.0841 \times 10^{6}\ m/s$$

$$V_{y,1} = V_{He}^{CM} \sin\theta_{cm,1} = 1.2032 \times 10^{6}\ m/s$$

Laboratory velocity for He_1 →

$$\boxed{V_{He_1} = \sqrt{(V_{x,1})^2 + (V_{y,1})^2} = 2.4065 \times 10^{6}\ m/s}$$

Which AGREES with part (a.)

What about the laboratory velocity for He_2 ?

$$V_{x,2} = V_{He}^{CM} \cos(233.79°) + V_{Be} = 3.220 \times 10^{5}\ m/s$$

$$V_{y,2} = V_{He}^{cm} \sin(233.79°) = -1.2032 \times 10^{6}\ m/s$$

Laboratory velocity for He_2 →

$$\boxed{V_{He_2} = \sqrt{(V_{x,2})^2 + (V_{y,2})^2} = 1.246 \times 10^{6}\ m/s}$$

which AGREES with part (a.)

1.16 Calculate the fraction of molecules with KE's between 0.02 kT and 0.04 kT $E_{midpoint} = 0.03\ kT$.

$$\text{fraction} \cong \frac{N(\epsilon)\,d\epsilon}{N} = \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} (0.03\,kT)^{1/2} e^{-0.03} (.02kT)$$

$$\boxed{\text{fraction} \cong 0.0037933}$$

Doing it exactly using Mathematica:

$$\text{fraction} = \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} \int_{0.02kT}^{0.04kT} \varepsilon^{1/2} e^{-\frac{\varepsilon}{kT}} d\varepsilon = \underline{\underline{0.0037733}}$$

Almost the same as the approximation above.

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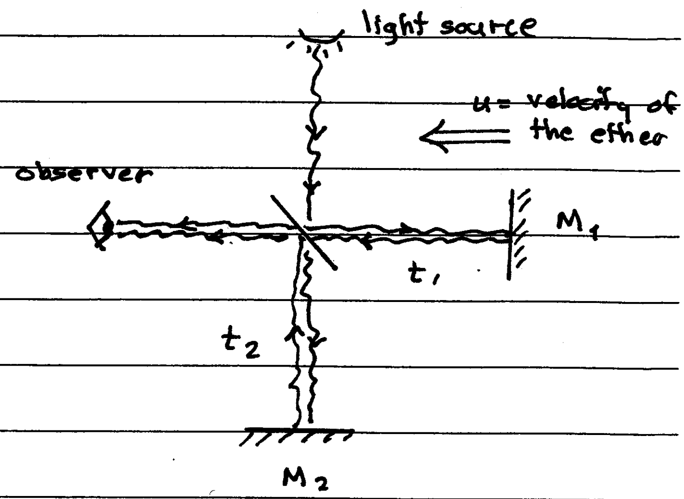
2.3 A shift of one fringe in the Michelson–Morley experiment

Δt = 1 period of light $\simeq 2\times10^{-15}$ sec when rotated 90°



$$t_1 = \frac{L}{c-u} + \frac{L}{c+u} = \frac{2Lc}{c^2-u^2}$$

$$t_1 = \frac{2L}{c}\,\frac{1}{1-u^2/c^2}$$

$$t_2 = \frac{L}{\sqrt{c^2-u^2}} + \frac{L}{\sqrt{c^2-u^2}} = \frac{2L}{\sqrt{c^2-u^2}}$$

$$t_2 = \frac{2L}{c}\,\frac{1}{\sqrt{1-u^2/c^2}}$$

$$\Delta t = t_1 - t_2 = \frac{2L}{c}\left[\frac{1}{\left(1-\frac{u^2}{c^2}\right)} - \frac{1}{\sqrt{1-u^2/c^2}}\right]$$

Using the Binomial Expansion, we obtain:

$$\Delta t = \frac{2L}{c}\left[\left(1+\frac{u^2}{c^2}+\dots\right) - \left(1+\frac{1}{2}\frac{u^2}{c^2}+\dots\right)\right]$$

$$\Delta t = \frac{2L}{c}\left[\frac{1}{2}\frac{u^2}{c^2}\right]$$ Now, solve for u

$$\frac{c\,\Delta t}{L} = \frac{u^2}{c^2} \qquad u = \sqrt{\frac{c\,\Delta t}{L}}\;c = \sqrt{\frac{3\times10^8\,(2\times10^{-15})}{11}}\;(3\times10^8\,\text{m/s})$$

$\boxed{u = 7.01\times10^4\ \text{m/s}}$ ← this would be the ether velocity

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2.7 The proper lifetime of a certain particle is 100 ns.

$V = 0.960\, c$ $\beta = 0.960$ $\gamma = \frac{1}{\sqrt{1-\beta^2}} = 3.5714$

$\tau_0 = 100$ ns.

lifetime in the lab.

a.) $\tau = \gamma \tau_0 = 3.57\,(100\text{ ns})$ $\boxed{\tau = 357\text{ ns}}$

b.) How far does it travel in the lab.

$L = \text{distance} = v\,\tau = 0.96\, c\, \tau$

$L = 0.960\,(3\times10^8\text{ m/s})\,(357\times10^{-9}\text{ s})$

$\boxed{L = 102.9\text{ meters.}}$

c.) How does that distance appear to an observer traveling with the particle?

In this context $L_0 = 102.9$ meters is the proper length so "L" is the quantity seen by the moving observer.

$$L = \frac{L_0}{\gamma} = \frac{102.9\text{ meters}}{3.5714} = \boxed{28.8\text{ meters}}$$