PS303 Exam #1

February 4, 2003

Answer Key

Name

10 points

1. An observer on a spacecraft moving at 0.700c relative to the earth finds that a car takes 40.0 min to make a trip. How long does the trip take for the driver of the car?
Answer in minutes

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.7^2}} = 1.40$$

$\tau = 40$ min

$\gamma \tau_0 = \tau \qquad \tau_0 = \tau/\gamma = \frac{40 \text{ min}}{1.40} =$ **28.6 minutes**

10 points

2. How much times does it take for a meter stick moving at 0.500c to pass an observer?
Answer in ns

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.5^2}} = 1.15$$

$L = v\tau_0 \qquad \tau_0 = \frac{L}{v} = \frac{L_0/\gamma}{v} = \frac{1.000 \text{ m}}{1.15 \;\; 0.5c} = 5.77 \times 10^{-9}$ s

$\tau_0 = 5.80$ ns

10 points

3. A spacecraft traveling 0.1c away from the earth sends a He-Ne laser beam whose "proper" wavelength is $\lambda_o = 632.8$ nm. What wavelength is measured by an observer on the earth?
Answer in nm

$$\nu = \nu_0 \sqrt{\frac{1-v/c}{1+v/c}} \Rightarrow \frac{c}{\lambda} = \frac{c}{\lambda_0}\sqrt{\frac{1-v/c}{1+v/c}} \qquad \lambda = \lambda_0 \sqrt{\frac{1+v/c}{1-v/c}}$$

$$\lambda = 632.8 \text{ nm}\sqrt{\frac{1+0.1}{1-0.1}} = 632.8 \text{ nm}\sqrt{\frac{1.1}{0.9}} =$$ **699.6 nm**

10 points

4. Find the momentum of a proton whose speed is 0.8c. **Answer in MeV/c**

$$p = mv = m_o \gamma v = m_o \gamma \beta c = m_o c \, \beta\gamma = 938 \frac{MeV}{c^2} \, c \, (0.8) \, 1.67$$

$$\boxed{p = 1251 \text{ MeV/c}}$$

20 points

5. A ρ^o meson at rest in the laboratory decays into two charged pions, $\rho^o \rightarrow \pi^+ \, \pi^-$. If the momentum of each pion in the laboratory is 358 MeV/c and the mass of a pion is 140 MeV/c^2:

   a. Calculate the total energy of each pion in the laboratory frame. **Answer in MeV**

$$E = \sqrt{p^2c^2 + m_o^2c^4} = \sqrt{358^2 + 140^2} = \boxed{384.4 \text{ MeV}}$$

   b. Using conservation of energy, what is the total energy of the ρ^o? **Answer in MeV**

$$E_{\rho^o} = E_{TOT} = E_{\pi^+} + E_{\pi^-} = \boxed{768.8 \text{ MeV}}$$

   c. Knowing the total energy of the ρ^o, what is the mass of the ρ^o? Recall that the ρ^o is initially at rest. **Answer in MeV/c^2**

$$E_{\rho^o} = m_{\rho^o} c^2 \qquad \boxed{m_{\rho^o} = 768.8 \text{ MeV/c}^2}$$

   d. What is the velocity of each pion in the laboratory frame? **Answer in units of c**

$$\beta = \frac{pc}{E} = \frac{(358 \text{ MeV/c})\, c}{\sqrt{p^2c^2 + m_\pi c^2}} = \frac{358 \text{ MeV}}{\sqrt{358^2 + 140^2}} = 0.93$$

$$\boxed{v = 0.93\, c}$$