Preparation for Exam # 1 for Modern Physics

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1 Chapter 1 Material

You should know the ideal gas law:

$$PV = Nk_BT$$

where P is measured in *pascals*, k_B is Boltzmann's constant (given) and T is the temperature in kelvin. The volume V is in m^3.

You should know how to calculate the *mean free path* between collisions:

$$\ell = \frac{1}{n\sigma}$$

where ℓ is the mean-free-path (nm), and $n = N/V$ is the number of atoms (or molecules) per unit volume, and σ is the cross-sectional area of the atom (or molecule). The diameter of an O_2 or N_2 molecule is typically 0.3 nm. Recall that the diameter of the hydrogen atom is ~ 0.1 nm. So, if we had a gas of O_2 molecules, the cross-sectional area for a single O_2 molecule is roughly $\sigma = \pi D^2/4$, or about $\frac{3}{4}D^2$ or ~ 0.0675 nm^2. The reason I want you to know how to calculate the *mean free path* is because it shows up on the Physics GRE.

As an exercise, calculate the mean free path of oxygen molecules at room temperature (20^oC). Assume the pressure is 1.00 atmosphere and the *diameter* of the oxygen molecule is 0.3 nm. Use the ideal gas law to determine n. **Answer:** 564 nm

2 Special Relativity – Kinematics

You should know the difference between Galilean transformations and Lorentz transformations when it comes to space, time, and velocity transformations.

2.1 Lorentz Transformation

All the equations used in this section assume that the origins of the two inertial frames (O and O') coincide at $t = t' = 0$. We also assume that the *moving* inertial frame, whose origin is O', is moving at constant velocity $V = \beta c$.

We derived in class the following Lorentz transformations for *space* and *time*:

$$x' = \gamma\,(x - Vt) \qquad t' = \gamma\left(t - \frac{V}{c^2}x\right) \quad \text{or} \quad ct' = \gamma\,(ct - \beta x) \tag{1}$$

The inverse Lorentz transformations are:

$$x = \gamma\left(x' + Vt'\right) \qquad t = \gamma\left(t' + \frac{V}{c^2}x'\right) \quad \text{or} \quad ct = \gamma\left(ct' + \beta x'\right) \tag{2}$$

So, if we are given space-time coordinates (x, t) in the O frame, we can use Eq. 1 to calculate the space-time coordinates (x', t') measured in the O' frame.

The velocity transformation was derived using the equations found in Eq. 1 above:

$$v_x' = \frac{v_x - V}{1 - \frac{\beta v_x}{c}} \tag{3}$$

Likewise, the inverse velocity transformation is:

$$v_x = \frac{v_x' + V}{1 + \frac{\beta v_x'}{c}} \tag{4}$$

2.2 Some kinematic short-cuts

Using Eqs. 1 and 2, it can be shown that:

- An object with proper length L_o moving in our "O frame" is measured to be *shorter* according to the Lorentz *length contraction*

  $$L = \frac{L_o}{\gamma}$$

  where L is the length measured in our "O frame."

- A clock moving in our "O frame" appears to advance slower than the clocks in our inertial frame, and this is described by Lorentz *time dilation*

  $$\tau = \gamma\,\tau_o$$

  where τ is the time interval measure in our "O frame."

2.3 Space-Time Diagrams

- You should understand how to draw space-time diagrams as shown in my lecture notes and my homework solutions.
- You should know how to calculate *space* and *time* intervals (Δr and Δt) between two events and how they appear to observers in another inertial frame ($\Delta r'$ and $\Delta t'$) using space-time invariants:

$$(\Delta s)^2 = c^2 (\Delta t)^2 - (\Delta r)^2$$
$$(\Delta s')^2 = c^2 (\Delta t')^2 - (\Delta r')^2$$

Recall that the space-time interval Δs between two events is independent of the inertial frame, and can be described by the following invariant equation: $(\Delta s')^2 = (\Delta s)^2$.

2.4 Doppler Shift

By comparing the phases of electromagnetic waves ($kx - \omega t$ and $k'x' - \omega' t'$) observed between two inertial frames, we were able to derive the Doppler shift. Let's assume an electromagnetic wave having wavelength λ is emitted from a source at rest in our O frame, and is measured by an observer in a moving frame ($V = \beta c$), let's say in the O' frame. The wavelength measured by an observer in the O' frame (λ') is red-shifted if the observer is moving away from the source.

$$\lambda' = \lambda r \quad = \quad \lambda \sqrt{\frac{1+\beta}{1-\beta}} \qquad \text{(red-shifted)}$$

If the observer is moving toward the source with velocity ($V = \beta c$), then the observer in the O' frame measures a wavelength (λ') that is blue-shifted:

$$\lambda' = \frac{\lambda}{r} \quad = \quad \lambda \sqrt{\frac{1-\beta}{1+\beta}} \qquad \text{(blue-shifted)}$$

Wavelengths that are red-shifted play a critical role in determining the distance to far-away galaxies. A common parameter used in astrophysics is z, the red shift. It is defined by the following equation:

$$z = r - 1$$

The red-shift z is related to the distance d described by the following equation:

$$z = \frac{H_o}{c}\, d$$

where H_o is the *Hubble constant* and is 68 (km/sec)/Mpc.
Note: 1 pc = 3.26 light-years.

3 Special Relativity – Dynamics

$$p = m_o c\,\beta\gamma \qquad \text{Relativistic Momentum}$$

$$K \,=\, m_o c^2\,(\gamma - 1) \qquad \text{Relativistic Kinetic Energy} \tag{5}$$

If the potential energy is non-existent, or it can be ignored, then the following equations are useful:

$$E \,=\, mc^2 \,=\, m_o\,\gamma\,c^2 \qquad \text{Total Relativistic Energy}$$

$$E \,=\, \sqrt{p^2c^2 + m_o^2c^4} \qquad \text{Total Relativistic Energy (v.2)}$$

$$E \,=\, K \,+\, m_o c^2 \qquad \text{Total Relativist Energy (v.3)}$$

3.1 Short Cuts

$$\beta \,=\, \frac{pc}{E} \tag{6}$$

$$\gamma \,=\, \frac{E}{m_o c^2} \tag{7}$$

3.2 When is a particle considered to be relativistic?

The best way to determine whether or not a particle should be treated as relativistic, use Eq. 5 from above to determine its γ factor:

$$\gamma \,=\, 1 \,+\, \frac{K}{m_o c^2} \tag{8}$$

If γ is greater than 1.02, then there will be $>2\%$ difference (or error) incurred when using the non-relativistic equations (e.g., $p = m_o v$, $K = \frac{1}{2} m_o v^2$, etc.) We will make this our rule-of-thumb for determining whether or not a particle should be treated relativistically.

3.3 Binomial Expansion

Everyone should know the binomial expansion and how to use it when $x \ll 1$:

$$(1+x)^n \,=\, 1 \,+\, nx \,+\, \frac{n(n-1)}{2!}x^2 \,+\, \frac{n(n-1)(n-2)}{3!}x^3 \,+\, \cdots$$

3.4 Two-body Decays

When a particle of mass M is *at rest* and decays into two particles having masses m_1 and m_2, then the momentum of each of the two particles 1 and 2 can be calculated using the following equation:

$$p^2 = \frac{\left[M^2 - (m_1 + m_2)^2\right]\left[M^2 - (m_1 - m_2)^2\right] c^2}{4M^2} \qquad \text{(2-body decay)} \tag{9}$$

Many of you had problems making calculation using this equation. Recall that masses are in units of MeV/c^2, and momenta are in units of MeV/c. The units of p^2 in the above equation are MeV^2c^2. You need to take the square-root of both sides to get units of momentum, namely (MeV/c).

Also, you derived this equation in one of your homework sets. Once you know the particle's momentum p (and you know its mass m_o), you can calculate the relativistic factors β and γ, using $\beta = pc/E$ and $\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$. You can also use the relationship

$$\beta\gamma = \frac{p}{m_o c} \tag{10}$$

square both sides and solve for β.

4 Calculating Threshold Energies

A common activity at high-energy accelerators is the production of exotic particles in order to study their physical properties. This can be done by converting the kinetic energy of a proton in a "proton beam" into mass energy $m_X c^2$, where m_X is the mass of the exotic particle. For example, one may want to create some particles known as pi-zeros (π^o's). If protons in a "proton beam" collide with protons in a target, they can produce π^o's by the following inelastic reaction:

$$p\,p \;\rightarrow\; p\,p\,\pi^o$$

where the first "p" in the equation usually refers to the beam particle, and the second "p" refers to the proton in the target (in a fixed-target experiment).

In order to solve a problem like this, we need to conserve both momentum and total energy. This calculation can best be performed with the use of *energy-momentum* 4-vectors,

$$(P^\mu P_\mu)_{\text{lab,before}} = (P^\mu P_\mu)_{\text{CM,after}}$$

and solve for E_{beam}. Then one can use the relationship $K_{\text{beam}} = E_{\text{beam}} - m_p c^2$ to solve for K_{beam}. This is very similar to the homework problem you did in Homework #5. Look on my website to see how the solution was obtained.

Practice Problem: Assume a proton beam collides with protons in a fixed-target experiment to produce π^o's. What is the minimum kinetic energy required for the proton in order to produce a π^o? Assume the masses are the following: $m_p = 938.272$ MeV/c^2 and $m_{\pi^o} = 134.98$ MeV/c^2.
Answer: $K_{\text{beam}} = 280$ MeV

5 The exam

You need to bring the following items to the exam:

- You need a working calculator.
- You need a pen or pencil.
- This is a closed-book exam, and no notes or note cards are allowed.

I will provide the following:

- The exam
- Any constants you will need (e.g., Boltzmann's constant k_B, the rest-masses of particles and nuclei m_o, etc.)
- Scratch paper if you need it.

Partial credit is possible if you show your work on the exam. However, don't "spam the exam" with a lot of equations that have nothing to do with the problem. Also, you are expected to know basic physics principles you learned from previous physics courses like:

- Conservation of momentum
- Conservation of energy
- Distance = velocity $\times$ time
- ... etc. ...

Note: Knowing how to solve the homework problems is one of the best ways to study for my exams. You should also know how to solve the problems on the 3 old exams that are on my website.