**Exam #2  PS303  Modern Physics**
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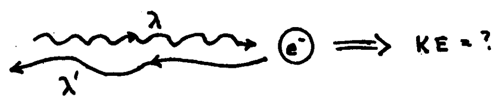
45 minutes

**Show your work !!**

Name: Answer Key

**20 points**

1. X-ray photons of kinetic energy $5.1113 \times 10^5$ eV are incident on "free electrons" at rest. After the interaction the "photons of interest" recoil at an angle of 180°.




a. What is the wavelength of the incoming photon?

$$\lambda = \frac{hc}{E} = \frac{1240\ \text{eV}\cdot\text{nm}}{5.1113 \times 10^5} = 2.426 \times 10^{-3}\ \text{nm}$$

$\lambda =$ $2.426 \times 10^{-3}$ nm

b. What is the wavelength of the photons recoiling at 180°?

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\phi) = \frac{2h}{m_e c} \qquad \lambda' = \lambda + \frac{2hc}{m_e c^2} = 2.426 \times 10^{-3}\ \text{nm} + 4.8532 \times 10^{-3}$$

$$\lambda' = 7.279 \times 10^{-3}\ \text{nm}$$

$\lambda' =$ $7.279 \times 10^{-3}$ nm

c. What is the energy of the recoiling photon?

$$E' = \frac{hc}{\lambda'} = \frac{1240\ \text{eV}\cdot\text{nm}}{7.279 \times 10^{-3}} = 1.7035 \times 10^5$$

$E' =$ $1.7035 \times 10^5$ eV

d. What is the kinetic energy of the electron that was initially at rest?

$$K = E - E' = 5.1113 \times 10^5 - 1.7035 \times 10^5$$
$$= 3.4078 \times 10^5$$

$KE =$ $3.4078 \times 10^5$ eV

e. Should the electron be considered relativistic, or non-relativistic. **Check one:**

☑ Relativistic   ☐ Non-Relativistic

$$\gamma = 1 + \frac{KE}{m_e c^2}$$
$$\gamma = 1.667$$

**10 points**

2. X-rays are produced when electrons are suddenly "braked" by their encounter with atoms in a target (i.e., bremsstrahlung). If the maximum-energy photon emitted in this process has a wavelength of 0.1000 nm,

a. What is the kinetic energy of the incoming electron?

$$K_{max} = \frac{hc}{\lambda_{min}} = \frac{1240\ \text{eV}\cdot\text{nm}}{0.1000\ \text{nm}} = 12{,}400$$

$KE_{electron} =$ 12.400 keV

b. What potential difference is required to accelerate the electrons?

$$U = qV_o = eV_o = KE$$

$$eV_o = 12{,}400 \text{ eV} \Rightarrow V_o = 12.4 \text{ kilovolts}$$

$\Delta V =$ 12.4 kilovolts

**5 points**

3. The temperature of the photosphere of Sirius is 9400K.
Calculate the intensity of light on the surface of the star.

$$I = \sigma T^4 = 5.67\times10^{-8}(9400\ K)^4 = 4.43\times10^{8}\ \frac{W}{m^2}$$

$I =$ $4.43\times10^{8}$ W/m²

**10 points**

4.

(a) Calculate the uncertainty in momentum for a proton confined to a nucleus of **radius** 6.0 fm.

$$\Delta x = 12.0 \text{ fm}$$

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{\hbar c}{\Delta x}\frac{1}{c} = \frac{197 \text{ MeV}\cdot\text{fm}}{12 \text{ fm}}\frac{1}{c}$$

$$\Delta p_x = 1.64\times10^{1}\ \frac{\text{MeV}}{c}$$

$\Delta p_x =$ 16.4 MeV/c

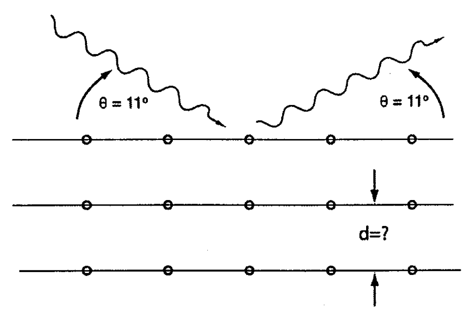
(b) What is the kinetic energy of a proton with that momentum?

$$K = \frac{(\Delta p_x)^2}{2m_p} = \frac{(\Delta p_x)^2 c^2}{2m_p c^2} = \frac{(16.4)^2 \text{MeV}^2}{2(938 \text{ MeV})} = 0.144 \text{ MeV}$$

$KE_{proton} =$ 0.144 MeV

**5 points**

5. X-rays having a wavelength of 0.124 *nm* undergo "first order" Bragg reflection at an angle of 11.0°. What is the spacing of adjacent planes in the crystal?




$$n\lambda = 2d\sin\theta$$

$$\lambda = 2d\sin\theta$$

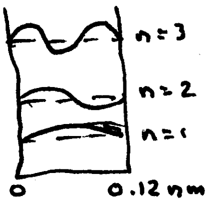
$$d = \frac{\lambda}{2\sin\theta} = \frac{0.124 \text{ nm}}{2(\sin 11^\circ)} = \frac{0.6499 \text{ nm}}{2}$$

$$d = 0.325 \text{ nm}$$

$d =$ 0.325 nm

**10 points**

6. An electron is confined to a one-dimensional infinitely-deep potential well having a width of 0.120 nm. Calculate the electron's change in kinetic energy when it makes a transition from the $n = 2$ state to the $n = 3$ state?




$$E_3 - E_2 = (9-4)\frac{\pi^2\hbar^2}{2mL^2} = \frac{5\pi^2(\hbar c)^2}{2m_e c^2 L^2} = \frac{5\pi^2(197\,eV\cdot nm)^2}{2(511{,}000\,eV)(0.120\,nm)^2}$$

$$\Delta K = E_3 - E_2 = 130.1$$

$\Delta K =$ 130.1 eV

**15 points**

7. An ensemble of quantum harmonic oscillators is described by the following wave function:

$$|\psi\rangle = A\,(\,2|0\rangle + 4|1\rangle + 4|2\rangle\,)$$

a. Calculate the normalization constant $A$ that will normalize this wavefunction.

$$\langle\psi|\psi\rangle = A^2(4 + 16 + 16) = 36|A|^2 = 1$$

$$A = \frac{1}{6}$$

$A =$ 1/6

b. Calculate the probability of finding a quantum harmonic oscillator in the $|1\rangle$ state.

$$\langle\psi|\psi\rangle = \frac{4}{36}\langle 0|0\rangle + \frac{16}{36}\langle 1|1\rangle + \frac{16}{36}\langle 2|2\rangle$$

$$\text{Prob.} = \frac{16}{36} = \frac{4}{9} = 0.4444\ldots$$

Prob = 44.4 %

c. What is the mean energy for this ensemble of quantum harmonic oscillators?

$$\langle E\rangle = |A|^2\left(2\langle 0| + 4\langle 1| + 4\langle 2|\right) E_{op}\left(2|0\rangle + 4|1\rangle + 4|2\rangle\right)$$

$$= \frac{1}{36}\left(2\langle 0| + 4\langle 1| + 4\langle 2|\right)\left(2\left(\tfrac{1}{2}\hbar\omega_0\right)|0\rangle + 4\left(\tfrac{3}{2}\hbar\omega_0\right)|1\rangle + 4\left(\tfrac{5}{2}\hbar\omega_0\right)|2\rangle\right)$$

$$= \frac{1}{36}\left[2\hbar\omega_0 + 24\hbar\omega_0 + 40\hbar\omega_0\right] = \frac{66}{36}\hbar\omega_0 = \hbar\omega_0$$

$\langle E\rangle =$ 1.83 $\hbar\omega_o$

**Useful Physical Constants:**

$P_o$ (1 atm) = $1.013\times10^5$ Pa
$1\,u = 1.661\times10^{-27}$ kg
$hc = 1240$ eV·nm
$m_e c^2 = 511{,}000$ eV

$k_B = 1.38\times10^{-23}$ J/K
$\sigma = 5.67\times10^{-8}$ W/(m²·K⁴)
$\hbar c = 197$ eV·nm
$m_p = 1836\,m_e$

$h = 4.14\times10^{-15}$ eV·s
$m_\mu = 207\,m_e$
$m_p = 938\,MeV/c^2$