PS303 Modern Physics Final Exam

April 28, 2015

Answer Key
Name

Closed Book, Closed notes. You can use a calculator

Show your work for partial credit. Multiple choice problems: circle the correct answer.

10 points

1. A container holds nitrogen gas (N_2) molecules (m = 28u) at a room temperature of 20° C and 1.00 atmosphere of pressure.

   a. Calculate the root-mean-square velocity of the gas molecules.

$$V_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3(1.38\times10^{-23})\ 293\,K}{1.6606\times10^{-27}\ 28}} = 511\ m/s$$

v_{rms} = 511 m/s

   b. Calculate the average kinetic energy of a single N_2 gas molecule under these conditions.

$$\bar{K} = \frac{3}{2}k_BT = \frac{3}{2}(1.38\times10^{-23})\,293\left(\frac{1.602\times10^{-19}\ J}{eV}\right)^{-1} = 0.0379\,eV$$

$\bar{K}$ = 0.0379 eV

10 Points (2 pts for each question)

2. A star (assumed to be at rest relative to the Earth) is 100 light-years from Earth. An astronaut sets out from Earth on a journey to the star at a constant speed of 0.986 c) $\gamma = 6.00$

   a. How long does it take for a light signal from Earth to reach the star, according to an observer on Earth?

   (1) 98 y (2) 100 y (3) 102 y (4) 16.7 y

   b. How long does it take for the astronaut to travel from Earth to the star, according to an observer on Earth? $L = v\tau$ $\tau = \frac{L}{v} = \frac{100\ c\cdot yrs}{0.986\,c} = 101.4$ y.

   (1) 16.7 y (2) 98 y (3) 100 y (4) 101.4 y (5) 105 y

   c. According to the astronaut, what is the distance from Earth to the star? $L = L_0/\gamma = \frac{100}{6} = 16.7$ l.yrs.

   (1) 100 l.y. (2) 101.4 l.y. (3) 20 l.y. (4) 16.7 l.y.

   d. According to the astronaut, how long does it take for the astronaut to travel from Earth to the star? $\tau = \gamma\tau_0$ $\tau_0 = \frac{\tau}{\gamma} = \frac{101.4}{6} = 16.9$ yrs.

   (1) 16.0 y (2) 16.7 y (3) 20 y (4) 100 y (5) 102 y

   e. Given your answers in (a) and (d), does this mean that the astronaut travels faster than light?

   (1) Yes (2) No (3) Maybe

5 points

3. A π^+ meson has a proper lifetime of 2.60×10^{-8} s. It is moving through the laboratory at a speed of $\frac{\sqrt{3}}{2}\,c$. What distance does the particle travel in the laboratory?

$\gamma = \frac{1}{\sqrt{1-3/4}} = 2$

$L = \beta \gamma c \tau_o = \frac{\sqrt{3}}{2}\ 2\ (3.00\times10^{8}\ m/s)\ 2.60\times10^{-8}\ s.$

$L = 13.5$ meters.

$L =$ 13.5 meters

10 points

4. The most intense radiation emitted from a hot sample of metal has a wavelength of 4.00μm.

a. If the sample absorbs and emits radiation like a blackbody ($\epsilon = 1$), what is the temperature of the sample?

$\lambda_{max} T = 2.90\times10^{-3}\ m\cdot K$ $\quad T = \frac{2.90\times10^{-3}\ m\cdot K}{4\times10^{-6}\ m} = 725$

$T =$ 725 K

b. How much power is radiated from the sample if its temperature is doubled, and the sample is in the shape of a cube 5.00 cm on a side?

$T = 1450$ K

$Power = \epsilon \sigma T^4 A = 1\left(5.67\times10^{-8}\ \frac{W}{m^2K^4}\right)\left(1450\right)^4\ 6\times\left(25\times10^{-4}\ m^2\right) = 3.76\times10^{3}$ watts.

$Power =$ 3.76 kW

5 points

5. Electrons are accelerated through a potential difference of 2000 volts and are incident on a metal surface, resulting in the emission of photons. Which of the following photon wavelengths would NOT be observed from this surface?

$K = 2{,}000\ eV = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{K} = \frac{1240\ eV\cdot nm}{2{,}000\ eV} = 0.62\ nm$ $\quad \boxed{\lambda_{min} = 0.62\ nm}$

(a) 0.24 nm (b) 0.78 nm (c) 1.25 nm (d) 3.62 nm

5 points

6. What is the kinetic energy of a proton whose deBroglie wavelength is 15.4 fm?

$p = \frac{h}{\lambda}$ $\quad pc = \frac{hc}{\lambda} = \frac{1240\ MeV\cdot fm}{15.4\ fm} = 80.5\ MeV$ $\quad \beta = \frac{pc}{E} = \frac{80.5}{941.44} = 0.085$

Non-relativistic $\Rightarrow K = \frac{p^2c^2}{2m_pc^2} = \frac{(80.5\ MeV)^2}{2(938.3\ MeV)} = 3.45\ MeV$

(a) 80.5 MeV (b) 3.45 MeV (c) 6340 MeV (d) 6.90 MeV

5 points

7. What is the de Broglie wavelength of an electron with a kinetic energy of 12.8 eV?

$\gamma = 1 + \frac{K}{m_e c^2} = 1 + \frac{12.8}{511{,}000} \Rightarrow$ non-relativistic

$p = \sqrt{2mK} \Rightarrow pc = \sqrt{2 m_e c^2 K} = \sqrt{2(511{,}000\,eV)\,12.8\,eV} = 3617\ eV$

$\lambda = \frac{hc}{pc} = \frac{1240\ eV\cdot nm}{3617\ eV} = 0.343\ nm$

(a) 96.9 nm (b) 0.34 nm (c) 16.8 nm (d) 0.66 nm

5 points

8. Experiments to measure the rest energy of a highly unstable elementary particle give a distribution of values centered at 2500 MeV with a width of ±40 MeV. Assuming the width is not due to any defect in the measuring instrument, what is the best estimate for the lifetime of the particle?

$\Delta E\ \Delta t > \hbar$ $\Delta t > \frac{\hbar}{\Delta E} = \frac{6.538 \times 10^{-16}\ eV\cdot s}{40\ MeV} = 1.63 \times 10^{-23}\ s$

(a) $10^{-19}\ s$ (b) $10^{-21}\ s$ (c) $10^{-23}\ s$ (d) $10^{-25}\ s$

5 points

9. A particle in the first excited state of a one-dimensional infinite potential energy well (with U = 0 inside the well) has an energy of 6.0 eV. What is the energy of this particle in the ground state?

$4 E_1 = 6.0\ eV$ $E_1 = \frac{6.0\ eV}{4} = 1.50\ eV$

(a) 1.0 eV (b) 1.5 eV (c) 2.0 eV (d) 3.0 eV

5 points

10. In a certain infinite potential energy well, the particle has a ground-state energy of 2.0 eV. Which of the following is NOT a possible value for the energy of one of the excited states of this particle in the well?

$E_1 = 2.0\ eV$

$E_2 = 4 E_1 = 8\ eV$

$E_3 = 9 E_1 = 18\ eV$

$E_4 = 16 E_1 = 32\ eV$

$E_5 = 25 E_1 = 50\ eV$

(a) 36 eV (b) ~~50 eV~~ (c) ~~18 eV~~ (d) ~~8 eV~~

5 points

11. An electron in the ground state of an infinite potential energy well has an energy of 8.0 eV. <u>How much additional energy</u> must be supplied for the electron to jump from the ground state to the first excited state?

$E_1 = 8.0\ eV$

$E_2 = 4(8.0\,eV) = 32\ eV$

$E_1 + E = E_2$ $E = E_2 - E_1 = 32\ eV - 8.0\ eV = 24.0\ eV$

(a) 8.0 eV (b) 16.0 eV (c) 24.0 eV (d) 32.0 eV

15 points

12. A neutral atom of the element boron (B) has 5 electrons. Four of the electrons are removed, forming an ion with the one remaining electron.

(a) Sketch an energy level diagram for this ion, showing the ground state and the first two excited states. Label each state with the value of its energy.

$$E_n = \frac{-E_R Z^2}{n^2} = \frac{(-13.6\ eV)\,5^2}{n^2}$$

$$E_n = -\frac{340\ eV}{n^2}$$

$E_1 =$ <u>-340</u> eV

$E_2 =$ <u>-85</u> eV

$E_3 =$ <u>-37.8</u> eV

$E_3 = -37.8\ eV$

$E_2 = -85\ eV$

$E_1 = -340\ eV$

(b) Calculate the two longest wavelengths at which this ion can absorb radiation. Assume all absorption occurs from the ground state.

$\to 2$

$$E_1 + \frac{hc}{\lambda} = E_2 \qquad \frac{hc}{\lambda} = E_2 - E_1 \qquad \lambda = \frac{hc}{E_2 - E_1} = \frac{1240\ eV\cdot nm}{255\ eV} = 4.86\ nm$$

$1 \to 3$

$$E_1 + \frac{hc}{\lambda} = E_3 \qquad \lambda = \frac{hc}{E_3 - E_1} = \frac{1240\ eV\cdot nm}{302.2\ eV} = 4.10\ nm$$

$\lambda_{1\to 2} =$ <u>4.86</u> nm

$\lambda_{1\to 3} =$ <u>4.10</u> nm

(c) What is the minimum amount of energy needed to remove the electron from the ground state of this ion?

$E_{min} = -E_1 = 340\ eV$

$E_{min} =$ <u>340</u> eV

5 points

13. In Rutherford scattering, what is the approximate ratio of the number of particles scattered at 7.1° to the number scattered at 4°?

$$\frac{N(7.1^\circ)}{N(4^\circ)} \sim \frac{\sin^4 4^\circ}{\sin^4 7.1^\circ} \cong \frac{1}{10}$$

(a) 1/2 (b) 1/4 (c) 1/10 (d) 1/16 (e) 1/20

15 points

14. A proton (p) and μ^- lepton are orbiting around each other in the ground state.

a. Calculate the reduced mass (m) for this system. $m_\mu = 207\, m_e$

$$m = \frac{m_p \cdot m_\mu}{m_p + m_\mu} = \frac{m_\mu}{1 + \frac{m_\mu}{m_p}} = \frac{207\ m_e}{1 + \frac{207}{1836}} = 186\ m_e$$

$m =$ 186 m_e

b. Calculate the radius of the orbiting muons in the "ground state."

$$r_n = \frac{n^2 \hbar}{m_e \alpha c} = \frac{n^2 \hbar}{186\ m_e \alpha c} = \frac{n^2}{186} a_0$$

$$n = 1 \text{ (ground state)} = \frac{1}{186} a_0 = 5.38 \times 10^{-3}\ a_0$$

$r_1 =$ 5.38×10^{-3} a_o

c. Calculate the "ground state" energy of this muonic atom.

$$E_n = -\frac{1}{2} m c^2 \frac{\alpha^2}{n^2} = \frac{-186}{n^2}\left(\frac{1}{2} m_e c^2 \alpha^2\right)$$

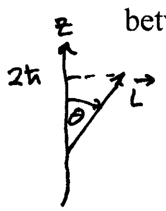
$$E_n = \frac{-186\,(E_R)}{n^2} \qquad E_1 = -186\,(13.6\,eV)$$

$$E_1 = -2530$$

$E_1 =$ -2530 eV

5 points

15. If an angular momentum vector has a maximum z component of $+2\,\hbar$, what is the angle between the angular momentum vector $\vec{L}$ and the z-axis?



$$|\vec{L}| = \sqrt{\ell(\ell+1)}\ \hbar = \sqrt{6}\ \hbar \qquad \cos\Theta = \frac{2\hbar}{\sqrt{6}\hbar} \qquad \Theta = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right) = 35.26^\circ$$

(a) 15° (b) 30.5° (c) 35.3° (d) 45°

10 points

16. A particle is in a one-dimensional coulomb potential energy well in the domain $(0 \le x \le \infty)$ $U(x) = -\frac{e^2}{4\pi\epsilon_0 x}$: If the wave function of the particle is $\psi(x) = A\,x\,e^{-x/b}$,

In exam 3 you calculated the normalization constant A for this wave function in terms of the constant b, and found that $A = \left(2/_{b^{3/2}}\right)$ Calculate the quantity $\langle x^2 \rangle$ and write your answer in the appropriate units of b

$$\langle x^2 \rangle = \int_0^\infty x^2 \psi^*(x)\psi(x)\,dx = A^2 \int_0^\infty x^4 e^{-2x/b}\,dx = \frac{4}{b^3}\,\frac{4!}{(2/b)^5}$$

$$\langle x^2 \rangle = \frac{4\,(24)}{32}\, b^2 = \underline{3b^2}$$

$$\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{3b^2 - \tfrac{9}{4}b^2} = \sqrt{\tfrac{3}{4}}\ b$$

$$\boxed{\sigma = \sqrt{\tfrac{3}{4}}\ b}$$

$\langle x^2 \rangle =$ $3\ b^2$

Useful Constants:

$m_{proton} = 938.3$ MeV $\qquad m_{electron} = 0.511$ MeV $\qquad m_{photon} = 0$ MeV

$R = 8.31\, J/(mol \cdot K)$

$1\, u = 1.6606 \times 10^{-27}\, kg$

$1\, eV = 1.602 \times 10^{-19}\, J$

$k_B = 1.38 \times 10^{-23}\, J/K$

$\lambda_{max}\, T = 2.90 \times 10^{-3}\, m \cdot K$

$\sigma = 5.67 \times 10^{-8}\; W/(m^2 \cdot K^4)$

$c = 3.00 \times 10^{8}$ m/s

$h = 6.626 \times 10^{-34}\, J \cdot s$

$\hbar = 6.583 \times 10^{-16}\, eV \cdot s$

$f_{>\theta} = (nt)\, \pi b^2$

$b = \frac{z\,Z}{2\,K}\, \alpha\, \hbar c\; cot\left(\frac{\theta}{2}\right)$

$\alpha = \frac{e^2}{4\,\pi\epsilon_o\, \hbar c} = \frac{1}{137.036}$

$N(\theta) = \frac{nt}{4\,r^2} \left(\frac{z\,Z}{2\,K}\right)^2 \frac{(\alpha\, \hbar c)^2}{sin^4\left(\frac{\theta}{2}\right)}$

$\mu_B = 5.788 \times 10^{-5}\, eV/T$

$\int_0^{\infty} x^n\, e^{-cx}\, dx = \frac{n!}{c^{n+1}}$