Exam #3 PS303
April 21, 2015

Answer Key
Name

Show your work !!

5 points

1. In a certain Rutherford scattering experiment with alpha particles, the distance of closest approach between the alpha particles and the nucleus is r. If the kinetic energy of the alpha particles is doubled, what is the new distance of closest approach?

Circle the correct answer:

$$d = \frac{e^2}{4\pi\varepsilon_0 K}$$

$K \to 2K$, then $d \to d/2$

(a) 2r

(b) 4r

(c) r/2

(d) r/4

5 points

2. In Rutherford scattering, what is the approximate ratio of the number of particles scattered at 10° to the number scattered at 5°?

Circle the correct answer:

$$N(10^\circ) \sim \frac{1}{\sin^4(5^\circ)} \qquad N(5^\circ) \sim \frac{1}{\sin^4(2.5^\circ)}$$

$$\frac{N(10^\circ)}{N(5^\circ)} = \frac{\sin^4(2.5^\circ)}{\sin^4(5.0^\circ)} \approx \frac{1}{15.94}$$

(a) 1/4

(b) 1/8

(c) 1/16

(d) 1/64

5 points

3. In a scattering experiment, protons pass through a thin foil of silver (Z = 47). The probability to detect the scattered protons has the value *I* at an angle of 10.0° relative to the direction of the original beam of protons. At what angle is the scattering probability 0.1*I* ?

Circle the correction answer:

$$I \simeq \frac{1}{\sin^4(5^\circ)} \qquad 0.1\,I \approx \frac{1}{\sin^4(\theta/2)}$$

$$\frac{0.1\,I}{I} = \frac{\sin^4(5^\circ)}{\sin^4(\theta/2)} \qquad \sin^4\left(\frac{\theta}{2}\right) = 10\sin^4(5^\circ)$$

$$\sin\left(\frac{\theta}{2}\right) = \sqrt[4]{10}\,\sin 5^\circ = 0.15$$

$$\theta = 2\sin^{-1}(0.155) = 17.8^\circ$$

(a) 40.0°

(b) 5.6°

(c) 2.5°

(d) 17.8°

(e) None of these.

10 points

4. a. The Balmer series of lines emitted by doubly ionized lithium (Li^{++}), which has atomic number 3, consists of electron transitions that end at the first excited state. Find the longest wavelength of the Balmer series of doubly ionized lithium.

$n=3 \rightarrow n=2$ longest wavelength

$$E_3 = E_2 + \frac{hc}{\lambda} \qquad \frac{hc}{\lambda} = E_3 - E_2 \qquad \lambda = \frac{hc}{(E_3-E_2)Z^2} = \frac{hc}{E_R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)Z^2} = \frac{hc}{E_R \, 5/36 \, Z^2}$$

$$\lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{13.6\text{ eV} \; 5/36(9)} = \cancel{656.5} \; 72.94 \text{ nm}$$

$\lambda =$ 72.94 nm

b. Find the longest wavelength at which doubly ionized lithium in its ground state can absorb a photon. $n = 1 \rightarrow n =$ 2

$$E_1 + \frac{hc}{\lambda} = E_2 \qquad \frac{hc}{\lambda} = E_2 - E_1 = E_R\left(\frac{1}{1^2}-\frac{1}{2^2}\right)Z^2 \qquad \lambda = \frac{hc}{E_R\left(\frac{3}{4}\right)9}$$

$$\lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{13.6\text{eV}\left(\frac{3}{4}\right)9} = 13.51 \text{ nm}$$

$\lambda =$ 13.5 nm

15 points

5. A μ^- lepton and μ^+ lepton are orbiting around each other in the ground state.

a. Calculate the reduced mass (m) for this system. $m_\mu = 207\, m_e$

$$m = \frac{m_\mu \cdot m_\mu}{m_\mu + m_\mu} = \frac{m_\mu^2}{2m_\mu} = \frac{m_\mu}{2} = 103.5\, m_e$$

$m =$ 103.5 m_e

b. Calculate the radius of the orbiting muons in the "ground state."

$$r_1 = \frac{1^2 \hbar}{m \alpha c} = \frac{\hbar c}{mc^2 \alpha} = \frac{197 \text{ MeV}\cdot\text{fm} \; 137}{103(.511 \text{ MeV})} =$$

$$r_1 = \left(\frac{\hbar}{m_e \alpha c}\right)\Big/103.5 = a_0 / 103.5 = 9.66\times10^{-3}\, a_0$$

$r_1 =$ 9.66×10^{-3} a_o

c. Calculate the "ground state" energy of this peculiar atomic system.

$$E_1 = -\frac{1}{2}\frac{mc^2\alpha^2}{1^2} = -103.5\left(\frac{1}{2}m_e c^2\alpha^2\right) = -103.5\, E_R$$

$$E_1 = -103.5\,(13.6\text{eV}) = -1.41\times10^3$$

$E_1 =$ -1.41×10^3 eV

2 points

6. Which of the following can also be quantum numbers of an $\ell = 2$ electron in hydrogen?

(a) $m_\ell = ½$ (b) n = 0 (c) n = 2 (d) $m_\ell = 0$

2 points

7. Which of the following is an allowed set of quantum numbers n, ℓ, m_ℓ, m_s for an electron in a hydrogen atom?

(a) 3,2,3,1/2 (b) 3,3,2,-1/2 (c) 3,1,0,-1/2 (d) 2,1,1,0

2 points

8. Which of the following sets of quantum numbers n, ℓ, m_ℓ, m_s is not allowed for an electron in a hydrogen atom?

(a) 2,0,0,+1/2 (b) 3,2,-2,-1/2 (c) 3,1,1,-1/2 (d) 2,2,0,1/2

2 points

9. If an angular momentum vector has a maximum z component of $+3\hbar$, how many different z components can it have?

multiplicity $= (2\ell+1) = 7$, $\ell = 3$

(a) 7 (b) 6 (c) 5 (d) 3

2 points

10. An electron is in an n = 2 state in a hydrogen atom. Which of the following can also be quantum numbers that describe that state of the electron?

(a) $\ell = 2,\ m_\ell = 0$ (b) $\ell = 1,\ m_\ell = +\frac{1}{2}$ (c) $\ell = -1,\ m_\ell = 0$ (d) $\ell = 1,\ m_\ell = -1$

10 points

11. A particle is in a one-dimensional coulomb potential energy well in the domain $(0 \le x \le \infty)$ $U(x) = -\frac{e^2}{4\pi\epsilon_0 x}$: If the wave function of the particle is $\psi(x) = A\,x\,e^{-x/b}$,

a. Calculate the normalization constant A for this wave function in terms of the constant b.

$$1 = A^2 \int_0^\infty x^2 e^{-2x/b}\,dx = A^2 \frac{2!}{(2/b)^3} = \frac{2A^2 b^3}{8} = 1 \qquad A^2 = \frac{4}{b^3}$$

$$A = \frac{2}{b^{3/2}}$$

$A =$ $A = 2/b^{3/2}$

b. Calculate the mean position $\langle x\rangle$ of the particle described by the wave function $\psi(x)$.

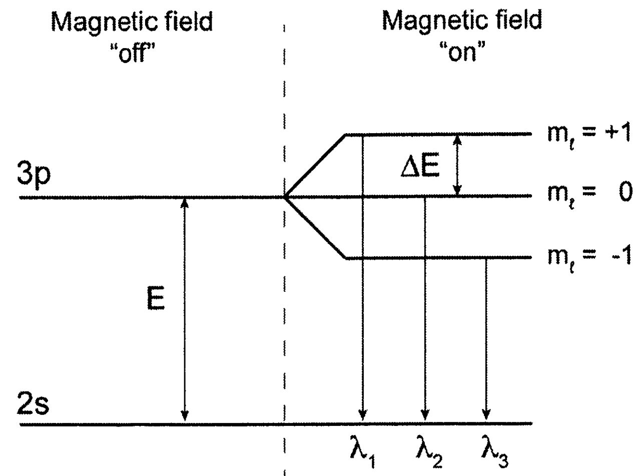
$$\langle x\rangle = A^2 \int_0^\infty x^2 e^{-2x/b}\, x\,dx = \frac{4}{b^3}\int_0^\infty x^3 e^{-2x/b}\,dx = \frac{4}{b^3}\frac{3!}{(2/b)^4}$$

$$\langle x\rangle = \frac{24}{b^3}\frac{b^4}{16} = \frac{3}{2}b$$

$\langle x\rangle =$ $3/2$ b

10 points

12. Hydrogen atoms are present in a uniform magnetic field of 1.5 tesla. The $n = 3 \rightarrow n = 2$ Balmer line is observed with three distinct wavelengths due to the Zeeman effect (ignoring spin effects). The splitting is shown in the figure.



a. Using the Bohr model, calculate the energy E.

$E_3 = E_2 + E \qquad E = E_3 - E_2$

$E = -E_R\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = E_R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36} E_R = \underline{1.89 \text{ eV}}$

E = 1.89 eV

b. Calculate the magnitude of the Zeeman splitting in the 3p state.

$\Delta E = \mu_B B = 5.788 \times 10^{-5} \frac{\text{eV}}{\text{T}} (1.5\,\text{T}) = 8.68 \times 10^{-5}$

$\Delta E =$ 8.68×10^{-5} eV

c. Calculate the separation in wavelength between the adjacent spectral lines $(\lambda_1, \lambda_2, \lambda_3)$.

$E = \frac{hc}{\lambda} \qquad \Delta E = \left|\frac{hc}{\lambda^2}\right| \Delta\lambda \qquad \Delta\lambda = \frac{\Delta E}{\frac{hc}{\lambda}\frac{1}{\lambda}} = \frac{\lambda\,\Delta E}{E}$

$\Delta\lambda = \left(\frac{hc}{E}\right)\frac{\Delta E}{E} = \frac{1240 \text{ eV}\cdot\text{nm}}{(1.89 \text{ eV})^2}\left(8.68 \times 10^{-5} \text{ eV}\right) = 0.0302 \text{ nm}$

$\Delta\lambda =$ 0.0302 nm

Useful Equations:

$$f_{>\theta} = (nt)\,\pi b^2 \qquad b = \frac{z\,Z}{2\,K}\,\alpha\,\hbar c\,\cot\left(\frac{\theta}{2}\right)$$

$$\alpha = \frac{e^2}{4\,\pi\epsilon_0\,\hbar c} = \frac{1}{137.036} \qquad N(\theta) = \frac{nt}{4\,r^2}\left(\frac{z\,Z}{2\,K}\right)^2 \frac{(\alpha\,\hbar c)^2}{\sin^4\left(\frac{\theta}{2}\right)}$$

$$\mu_B = 5.788 \times 10^{-5}\ eV/T \qquad \int_0^\infty x^n\,e^{-cx}\,dx = \frac{n!}{c^{n+1}}$$