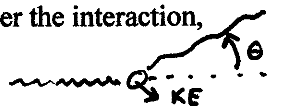
Exam #2 PS303 Modern Physics
Dr. Darrel Smith March 31, 2015

Show your work !! Answer Key
Name

10 points

1. X-ray photons of wavelength 0.01575 nm are incident on free electrons at rest. After the interaction, photons of wavelength 0.01772 nm are observed.



(a) Relative to the direction of the original X-rays, at what angle would we observe these photons?

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta) \qquad 1-\cos\theta = \frac{\lambda'-\lambda}{h/m_e c} \qquad \cos\theta = 1 - \frac{\lambda'-\lambda}{hc/m_e c^2}$$

$$\theta = \cos^{-1}\left[1 - \frac{0.01772\,nm - 0.01575\,nm}{1240\,eV\cdot nm/511{,}000\,eV}\right] = \cos^{-1}(0.1882) = 79.2^\circ$$

$\theta =$ 79.2 degrees

(b) What is the kinetic energy given to the electrons by this interaction?

$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + KE \qquad KE = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = 1240\,eV\cdot nm\left(\frac{1}{.01575} - \frac{1}{.01772}\right) = 8.75\,keV$$

$KE_e =$ 8.75 keV

10 points

2. An ultraviolet lamp of adjustable wavelength is shining on a metal surface. It is observed that electrons begin to emerge from the surface when the wavelength is 255 nm.

(a) What is the minimum energy necessary to remove an electron from the surface of this metal?

$$\lambda_{max} = 255\,nm \qquad E_{min} = \frac{hc}{\lambda_{max}} = \frac{1240\,eV\cdot nm}{255\,nm} = 4.86\,eV$$

$Emin =$ 4.86 eV

(b) If the wavelength is reduced to 215 nm, what is the energy of the electrons that leave the surface?

$$\frac{hc}{\lambda} = 4.86\,eV + KE \qquad KE = \frac{1240\,eV\cdot nm}{215\,nm} - 4.86\,eV = 0.905\,eV$$

$KE_e =$ 0.905 eV

10 points

3. In a certain experiment, the momentum of a beam of electrons is measured to have a value of 14.2 keV/c, with a spread in values of ±0.7 keV/c.

(a) What is the minimum size of the apparatus containing the electrons that is necessary to make this measurement?

$$\Delta x = \frac{\hbar c}{\Delta p c} = \frac{197\,eV\cdot nm}{700\,eV} = 0.2814\,nm$$

$\Delta x =$ 0.28 nm

(b) What is the deBroglie wavelength of these electrons?

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1240\ \text{eV}\cdot\text{nm}}{14{,}200\ \text{eV}} = 0.0873$$

$\lambda =$ 0.0873 nm

10 points

4.

(a) Calculate the uncertainty in momentum for a proton confined to a nucleus of radius 6.0 fm.

$$\Delta p_x = \frac{\hbar}{\Delta x} = \frac{\hbar c}{Dc} = \frac{197\ \text{MeV}\cdot\text{fm}}{12\ \text{fm}\ c} = 16.4\ \text{MeV}$$

↑Diameter

$\Delta p_x =$ 16.4 MeV/c

(b) What is the kinetic energy of a proton with that momentum?

$$K = \frac{(\Delta p_x)^2 c^2}{2m_p c^2} = \frac{(16.4\ \text{MeV}/c)^2 c^2}{2(938\ \text{MeV})} = 0.144\ \text{MeV}$$

$KE_{proton} =$ 0.144 MeV

(c) Suppose a proton in that nucleus had a kinetic energy of 5.6 MeV. If the proton were represented by a deBroglie wave, how many wavelengths could fit across the diameter of that nucleus?

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{hc}{\sqrt{2m_pc^2 K}} = \frac{1240\ \text{MeV}\cdot\text{fm}}{\sqrt{2(938)\ 5.6}} = 12.09\ \text{fm} \approx \text{Diameter} = 12.0\ \text{fm}$$

number of deBroglie waves = 1 (nearest integer)

10 points

5.

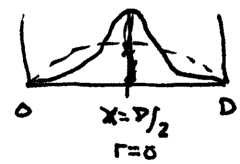
(a) A neutron ($mc^2 = 939.6$ MeV) is confined in a nucleus of diameter 11 fm. Inside the nucleus, the neutron moves freely (no forces act on it), but at the edges of the nucleus a very strong force (which we can take to be infinitely strong) prevents the neutron from leaving the nucleus. Treating this as a one-dimensional problem, find the energy difference between the ground state and the first excited state of the neutron.

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad \Delta E = E_2 - E_1 = \frac{(4-1)\pi^2\hbar^2c^2}{2m_nc^2L^2} = \frac{3\pi^2(197\ \text{MeV}\cdot\text{fm})^2}{2(939.6\ \text{MeV})(11\ \text{fm})^2}$$

$\Delta E = 5.05$ MeV

$\Delta E =$ 5.05 MeV

(b) In the ground state, what is the probability to find the neutron in a narrow region of width 0.10 fm located at the center of the nucleus? Do not integrate



$$\psi^*\psi\, dx = \frac{2}{L}\sin^2\frac{\pi x}{L}\, dx$$

$$\psi^*\psi\, dx = \frac{2}{L}\sin^2\frac{\pi}{2}\, dx = \frac{2}{\cancel{D}}\,\frac{0.10\text{fm}}{11\text{fm}}\,\cancel{D}$$

$$\psi^*\psi\, dx = \frac{0.20}{11\text{fm}} = 0.0182 = 1.82\%$$

probability = 1.82 %

10 points

6. An ensemble of quantum systems is prepared according to the following wave function:

$$\psi = A(\,2|1\rangle + 4|2\rangle + 3|4\rangle)$$

a. Find the normalization constant, A.

$$A = \frac{1}{\sqrt{4+16+9}} = \frac{1}{\sqrt{29}}$$

A = 0.1857

b. Find the probability of measuring a system in the $|2\rangle$ state.

$$\frac{16}{29} = 0.5517 \quad \text{or} \quad 55.2\%$$

probability = 55.2 %

Useful constants:

$hc = 1240\ eV \cdot nm$

$\hbar c = 197\ eV \cdot nm$

$m_e c^2 = 511{,}000$ eV

$m_p c^2 = 938$ MeV