PS303 **Exam #1** **Modern Physics**

February 18, 2013

Show your work!

Answer Key
Name

10 points

1. A meter stick moves parallel to its axis with speed 0.96 c relative to you.

   a. What would you measure for the length of the stick?

$$L = \frac{L_o}{\gamma} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.96)^2}} = 3.571$$

$$L = \frac{L_o}{\gamma} = \frac{1.000\ m}{3.571} = 0.280\ m$$

$L =$ 0.280 *meters*

   b. How long does it take for the stick to pass you?

$$L = v\tau \qquad \tau = \frac{L}{v} = \frac{0.280\ m}{0.96(3\times10^8\ m/s)} = 0.972\ ns$$

$\tau =$ 0.972 *ns* (10^{-9} *sec*)

10 points

2. The diameter of our galaxy is ~ 100,000 c-years. A spaceship sets out to cross our galaxy in 25 years, as measured on board the ship.

   With what uniform speed does the spaceship need to travel?

$$L = v\tau = v\gamma\tau_o = \beta\gamma c\tau_o \qquad \beta\gamma = \frac{L}{c\tau_o} = \frac{100{,}000\ c\cdot yrs}{c\ (25\ yrs)}$$

$$\beta\gamma = 4{,}000 \qquad \beta^2\gamma^2 = (4{,}000)^2 = x$$

$$\beta^2\frac{1}{1-\beta^2} = x \qquad \beta^2 = x(1-\beta^2) \Rightarrow \beta^2 + \beta^2 x = x \Rightarrow \beta^2(1+x) = x$$

$$\beta^2 = \frac{x}{1+x} = \frac{16{,}000{,}000}{16{,}000{,}001} \qquad \beta = 0.99999996875$$

$v =$ 0.999 999 968 75 *c*

10 points

3. A pole vaulter holds a 4.90 meter pole horizontal to the ground as he runs 9.00 m/s with respect to the track. By how much does the pole shrink as measured in the track's inertial frame?
 Hint: Use the binomial expansion to calculate $\Delta L = L_o - L$.

$$\Delta L = L_o - L = L_o - \frac{L_o}{\gamma} = L_o\left(1 - \frac{1}{\gamma}\right) = L_o\left(1 - \sqrt{1-\beta^2}\right)$$

$$\Delta L \cong L_o\left(1 - \left(1 - \frac{1}{2}\beta^2 + \cdots\right)\right) = L_o\left(\frac{1}{2}\beta^2 + \cdots\right) = 4.90\,m\left(\frac{1}{2}\left(3\times10^{-8}\right)^2\right)$$

$$\boxed{\beta = \frac{9.00\,m/s}{3\times10^{8}\,m/s} = 3\times10^{-8}}$$

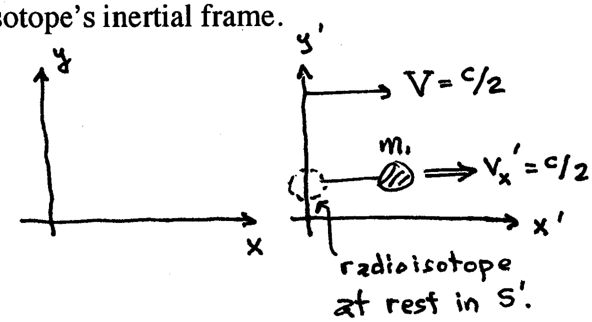
$$\Delta L = 2.205\times10^{-15}\,m$$

$\Delta L =$ 2.205 $fm\ (10^{-15}\ m)$

10 points

4. A radioisotope is moving in the $+x$ direction at a speed of c/2 as measured in the lab frame. While in motion, the radioisotope decays and one of the particles (m_1) is emitted at a speed of c/2 in the $+x'$ direction as measured in the radioisotope's inertial frame.

 a. What is the velocity of m_1 in the lab frame?



$$V_x = \frac{V_x' + V}{1 + \frac{\beta V_x'}{c}} = \frac{c/2 + c/2}{1 + \frac{1}{2}\frac{c}{2}\frac{1}{c}}$$

$$V_x = \frac{c}{5/4} \qquad \boxed{V_x = \frac{4}{5}c}$$

$v =$ 0.800 c

 b. What is the gamma-factor (γ) for m_1 as measured in our frame?

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1 - 16/25}} = \frac{1}{\sqrt{9/25}} = \frac{5}{3} = 1.67$$

$\gamma =$ 1.67

10 points

5. A particle with a rest-mass energy of 2.40 GeV (*1 GeV* = 10^9 *eV*) has a total energy of 15.0 GeV. Find the time (in the Earth's inertial frame) necessary for this particle to travel from Earth to a star 4.00 light-years away. **Hint:** Find the momentum, and then the velocity (β).

$E = 15.0$ GeV

$m_0c^2 = 2.40$ GeV

$$pc = \sqrt{E^2 - m_0^2c^4} = \sqrt{(15.0)^2 - (2.40)^2}\ \text{GeV}$$

$$pc = 14.8068\ \text{GeV}$$

$$\beta = \frac{pc}{E} = \frac{14.8068\ \text{GeV}}{15.0\ \text{GeV}} = 0.987 \qquad v = \beta c \qquad \boxed{v = 0.987\ c}$$

$$\tau = \frac{L}{v} = \frac{4.00\ c\cdot\text{yrs}}{0.987\ c} = 4.05\ \text{yrs}$$

τ = 4.05 *years*

15 points

6. While at rest, a rho-plus particle decays into two pions ($\rho^+ \to \pi^+ \pi^o$). What is the momentum of the decay products?

$m_{\rho^+}c^2 = 765$ MeV $\quad m_{\pi^+}c^2 = 140$ MeV

$m_{\pi^o}c^2 = 135$ MeV

Hint: Solve for the energy of one of the pions,

$$E_{\rho^+} = E_{\pi^+} + E_{\pi^o} \qquad E_{\pi^+} = E_{\rho^+} - E_{\pi^o} \Rightarrow E_{\pi^+}^2 = (E_{\rho^+} - E_{\pi^o})^2$$

$$p^2c^2 + m_{\pi^+}^2c^4 = E_{\rho^+}^2 - 2E_{\pi^o}E_{\rho^+} + E_{\pi^o}^2 = m_{\rho^+}^2c^4 - 2E_{\pi^o}m_{\rho^+}c^2 + p^2c^2 + m_{\pi^o}^2c^4$$

$$m_{\pi^+}^2c^4 = m_{\rho^+}^2c^4 - 2E_{\pi^o}m_{\rho^+}c^2 + m_{\pi^o}^2c^4 \Rightarrow E_{\pi^o} = \frac{m_{\rho^+}^2c^4 - m_{\pi^+}^2c^4 + m_{\pi^o}^2c^4}{2m_{\rho^+}c^2}$$

$$E_{\pi^o} = \frac{765^2 - 140^2 + 135^2}{2(765)} = 381.6\ \text{MeV}$$

then calculate the momentum of the pion.

$$pc = \sqrt{E_{\pi^o}^2 - m_{\pi^o}^2c^4} = \sqrt{(381.6)^2 - (135)^2} = 356.9\ \text{MeV}$$

$$\boxed{p = 357\ \text{MeV}/c}$$

p_π = 357 MeV/c