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Thermal Radiation Studies for an Electron-Positron Annihilation Propulsion System

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In this paper we describe a space propulsion system based on *electron-positron* annihilation. Two methods of propulsion are investigated. The first method is the direct absorption of photons from e^+e^- annihilation on a tungsten shield resulting in a thrust-to-mass ratio of 1.5×10^{-6} g's. However, this method requires many kilograms of antimatter and produces large amounts of thermal energy in order to produce a modest acceleration. The second method investigates how the thermal energy can be extracted (evaporative cooling) by heating liquid hydrogen and using it as a propellant. This technique results in a more substantial thrust-to-mass ratio of ≥ 0.043 g's and requires less than a gram of antimatter in the form of positrons.

Nomenclature

$\bar{p}$	antiproton, mass = 938 MeV/c^2
e^+	positron, mass = 0.511 MeVc^2
γ	photon (momentum = 0.511 MeV/c)
c	speed of light, 3.00×10^8 m/s
$\dot{m}$	rate of mass annihilation of positrons
σ	Stefan-Boltzmann constant (5.67×10^{-8} W/(m$^2\cdot$ K^4))
g	acceleration due to gravity (9.81 m/s^2)
ϵ	emissivity of tungsten (0.35)
ρ	density of tungsten (19.3 g/cm^3)
X_o	radiation length of tungsten (0.35 cm)
$\dot{m}_{\text{LH2}}$	the mass rate of liquid hydrogen used in evaporative cooling
h	convective heat transfer coefficient for LH2 (210 W/(m$^2\cdot$K))
C_{p}	specific heat at constant pressure for LH2 (14,304 J/(kg$\cdot$K))

I. Introduction

For years the space industry has been exploring the use of more powerful, more efficient propulsion systems to propel a manned spacecraft for interplanetary missions. Early in this search, it became clear that chemical propulsion systems had reached their maximum efficiencies and other exotic (i.e., non-chemical) propulsion systems should be investigated. In some sense, it was time to move from the low-energy atomic energy sources to higher-energy nuclear or particle physics sources.

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One of the *figures of merit* commonly used to describe the "momentum transfer" of various engines is the specific impulse (i.e., the momentum delivered compared to the weight of the propellant). A typical graph showing the specific impulses for different exotic propulsion engines is shown in Fig. 1. The maximum specific impulse obtained from *chemical* engines is ~500 s on this log-log plot. The remaining engines with higher specific impulse are all "nuclear" or "particle" in nature. Since the discovery of the positron (1933) and the antiproton (1955), the most efficient conversion of matter into energy imaginable is due to the annihilation of particle-antiparticle systems such as proton-antiproton ($p\bar{p}$) and electron-positron (e^-e^+) annihilations. When comparing the momentum transfers produced by these two annihilation processes, the first one produces massive final state particles ($p\bar{p} \rightarrow \pi^+\pi^-\pi^o + \ldots$) while the second one produces massless, high-energy photons ($e^-e^+ \rightarrow \gamma\gamma$), that is, the total energy of the e^-e^+ annihilation is converted into "pure" momentum with no energy converted into mass. The specific impulse for such a process is $\sim c/g$ or about 3×10^7 s, much higher than any of the other processes described in Fig. 1 save the Solar Sail.[1]

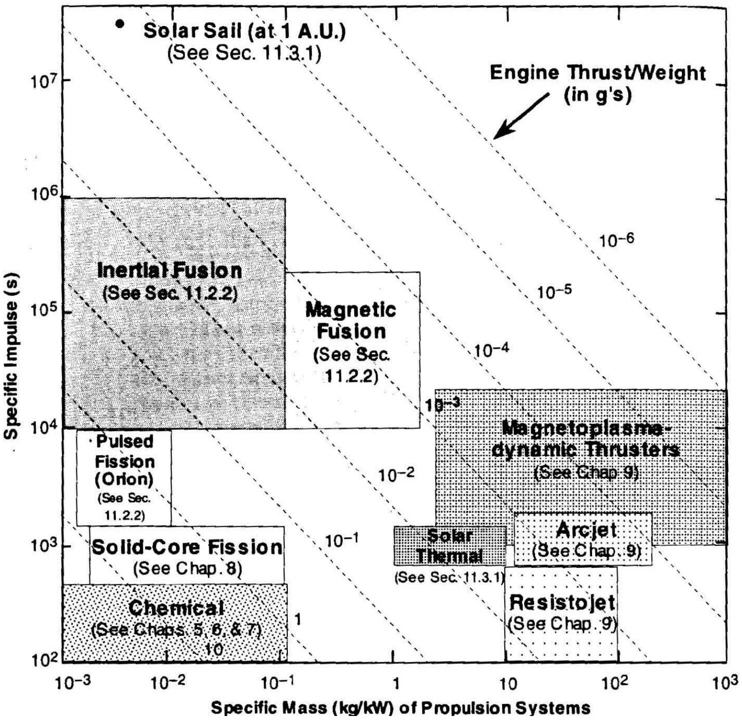

Figure 1. The specific impulse for various propulsion systems as a function of specific mass.[2]

II. Photon momentum transfer due to e^-e^+ annihilation

Approximately 99% of the e^-e^+ annihilations result in the production of two 0.511 MeV photons. The annihilation process occurs while "mixing" electrons and positrons at low energies. After the electrons and positrons form Bohr orbits lasting $\sim 10^{-10}$s, they annihilate into a pair of gamma rays.

In the simplest scenario, one can imagine that the gamma-ray momentum is directly transfered to a shield attached to the spacecraft. The shield material must be thick enough to stop 99% of the gamma-rays (i.e., 5 radiation lengths) and capable of maintaining its structural integrity at high temperatures. When searching for the optimum shield material a number of physical properties must be compared between the various candidates. Taking into account the melting temperature, density, and radiation length of platinum, lead, beryllium, and tungsten, tungsten proved to be the most promising candidate for the shield material. Figure 2 shows the photons produced at the center of a semi-spherical tungsten shell of radius R and thickness t.

The rate of momentum transfer to the shield is proportional to the rate of e^-e^+ annihilation $\dot{m}c$, and the angle subtended by the shield θ. In this geometry, one of the two photons impacts the surface radially,

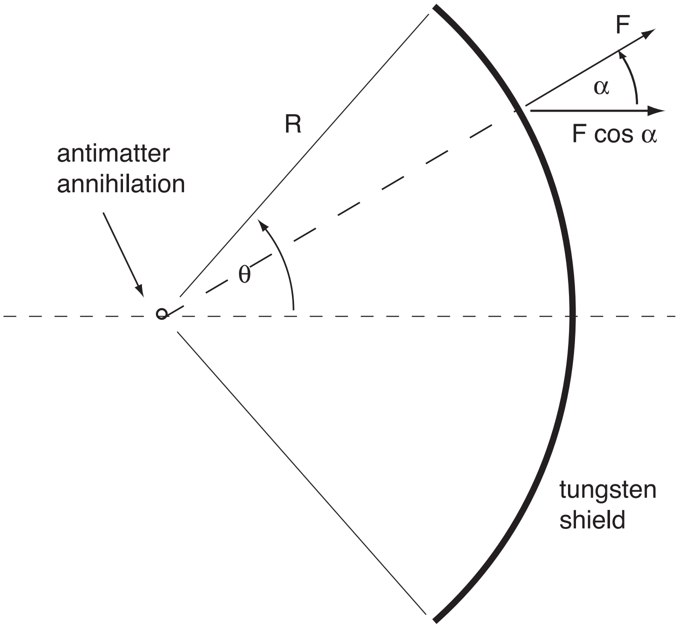

Figure 2. Photons from e^-e^+ annihilation incident upon a semi-spherical tungsten shield of radius R, opening angle θ and thickness t. A photon incident upon the shield at an angle α produces a component of thrust along the axis of symmetry of $F\cos\alpha$.

however, only the momentum component parallel to the axis of symmetry $p_{\|}$ contributes to the thrust of the spacecraft. The momentum perpendicular to the axis $p_{\perp}$ does not contribute to the thrust of the rocket but still contributes to the overall heating of the shield.

A. Thrust-to-mass of the shield

The force F due to the momentum transfer of gamma rays to the shield is proportional to the rate of antimatter annihilation $\dot{m}$, the geometrical acceptance of the semi-spherical shield of angle θ, and the mean $\cos\alpha$ over the semi-spherical surface,

$$F = \dot{m}c\,(\text{geometrical acceptance}) \times \langle\cos\alpha\rangle \;=\; \dot{m}c\,(1-\cos\theta)\left(\frac{1+\cos\theta}{2}\right) \tag{1}$$

where the geometrical acceptance is $1/2\pi\int_0^{2\pi}\int_0^{\theta} d\Omega$ and $d\Omega$ is the solid angle $\sin\theta'\,d\theta'\,d\phi$ where ϕ is the azimuthal angle around the axis of symmetry. Likewise, $\langle\cos\theta\rangle$ can be written as $(1/\Delta\Omega)\int_0^{2\pi}\int_0^{\theta}\cos\alpha\,d\Omega$ where $d\Omega = \sin\alpha\,d\alpha\,d\phi$. The mass of the shield is proportional to the density ρ, the thickness t, and the fraction of solid angle subtended by a semi-spherical shield of angle θ:

$$m_{\text{shield}} = 2\pi\rho R^2\,(1-\cos\theta)\,t \tag{2}$$

The thrust/mass (in g's) for the shield alone can be calculated from dividing Eq 1 by Eq. 2.

$$\text{Thrust/mass (in g's)} \;=\; \frac{\dot{m}c}{2\pi\rho R^2 t g}\left(\frac{1+\cos\theta}{2}\right) \tag{3}$$

B. Thermal considerations of the shield

At first glance Eq. 3 appears to provide an endless range of *thrust-to-mass*, however, the only method for discharging the accumulated thermal energy is by way of blackbody radiation

$$\underbrace{\dot{m}c^2(1-\cos\theta)}_{\text{energy absorbed}} \;=\; \underbrace{\epsilon\sigma T^4\left(2\pi R^2(1-\cos\theta)\right)}_{\text{energy radiated}} \tag{4}$$

where T (the melting point of the shield material) becomes the limiting factor. Solving Eq. 4 for $\dot{m}c$ and substituting it into Eq. 3, the thrust/mass (*in g's*) can be written in terms of the physical properties of the material used in the shield, namely, the emissivity, the density, the thickness, and the angle θ of the shield.

$$\text{Thrust/mass (in g's)} = \frac{\epsilon\sigma T^4}{\rho c t g}\left(\frac{1+\cos\theta}{2}\right) \tag{5}$$

To maximize the power emitted through blackbody radiation, tungsten is the obvious "material of choice" with its high melting temperature of 3500K. Furthermore, the thrust is maximized by using a hemisphere ($\theta = 90\,\text{deg}$). In order to stop 99% of the photons, a thickness of five radiation lengths ($5X_o$) of tungsten is used. For a completely absorbing shield, the thrust/mass (*in g's*) is 1.50×10^{-6}.

At first glance, the annihilation of positrons into "pure momentum" appears very promising, however, the thrust/mass ratio calculated from Eq. 5 is quite small. A factor of 2 could be gained by including radiation from both sides of the shield, however, this only raises the thrust/mass to $\sim 3\times10^{-6}$ g's, placing the *photon drive* off to the right of the Solar Sail in Fig. 1. However, the performance characteristics quoted for the Solar Sail in Fig. 1 are at a distance of 1 AU from the sun and deteriorate rapidly for missions to the outer planets, whereas, the *photon drive* maintains its performance characteristics at any location.

One way to avoid overheating the shield is to use a parabolic shield that totally reflects the incoming photons due to the positron-electron annihilation. The virtue of such a shield is obvious. This technique would substantially increase the thrust while providing a modest improvement to the specific impulse.[1] Unfortunately, the technology to reflect 0.511 MeV photons does not currently exist. However, if it did, it would still require many kilograms of antimatter to make this system practical.

Another variation of this model might include depositing the positrons directly on the semi-spherical shell instead of annihilating them at the center. At least half of the gamma-ray photons would contribute to the thrust, however, their directions are random and *heat* is still the limiting factor. If positrons are going to be relevant to future manned space flights, and the main byproduct of antimatter annihilation is thermal energy (and not momentum), the question to address is, "Can antimatter annihilation be used as an efficient heat source similar to the reactor core used in nuclear thermal propulsion?"

III. Positron Thermal Engine

In this section, we explore the thermal heating due to positron annihilation and its potential for creating higher *thrust/mass* ratios when compared to the *photon drive*. One possible method for enhancing the thrust is through evaporative cooling using liquid hydrogen (LH2). In order to dissipate the excess heat produced by photon absorption on the tungsten shield, a coolant could be uniformly applied to the superheated, hemispherical shield. The naturally occurring heat transfer will raise the liquid hydrogen (LH2) to the temperature of a superheated gas, at which point, it will expand out a rocket nozzle and provide substantial thrust.

A. Evaporative cooling of LH$_2$

In this scenario, the tungsten shield is uniformly heated due to positron annihilations on its surface so as to create a heat reservoir at a constant temperature of ~3300K. The rate of heat transfer from the shield to the LH2 can be written as

$$\dot{Q} = h\,A\,(T_{\text{shield}} - T_{\text{LH2}}) \tag{6}$$

where h is the convective heat transfer coefficient and A is the area of the shield. The temperature gradient between the shield (3300K) and the LH2 (16K) is fixed at 3284K while the convective heat transfer coefficient[3] for LH2 is 210 W/(m^2·K).

Since the thrust F is the product of the mass flow rate $\dot{m}$ and the exhaust velocity of the hydrogen gas v_{H_2}, both quantities must be calculated. The mass flow rate can be determined by balancing the thermal power leaving the tungsten $\dot{Q}$ with the mass flow rate of LH2 absorbing the thermal energy and this is described by the following equation:

$$\dot{m}_{\text{LH2}} = \frac{\dot{Q}}{C_{\text{p}}\,(T_{\text{shield}} - T_{\text{LH2}})} \tag{7}$$

where C_p is the specific heat at constant pressure for the LH2.[4] The mass flow rate of LH2 as a function of surface area is found by substituting Eq. 6 into Eq. 7.

$$\dot{m}_{_\mathrm{LH2}} = \frac{hA}{C_\mathrm{p}} \;=\; (0.021\,A)\ \mathrm{kg/s} \tag{8}$$

The exhaust velocity of the hydrogen gas $v_{_{\mathrm{H}_2}}$ can be determined by relating the change in kinetic energy due to the thermal power convectively absorbed from the tungsten shield. As the H_2 molecules absorb the thermal energy, the kinetic energy increases by an amount $E \sim \frac{1}{2} m_{_{\mathrm{H}_2}} v^2_{_{\mathrm{H}_2}}$ since the kinetic energy of hydrogen molecules is negligibly small at low temperatures. The velocity of the *heated* hydrogen molecules is found to be

$$v_{_{\mathrm{H}_2}} = \sqrt{\frac{2\,E}{m_{_{\mathrm{H}_2}}}} = \sqrt{\frac{2\dot{E}}{\dot{m}_{_{\mathrm{H}_2}}}} = \sqrt{\frac{2\dot{Q}}{\dot{m}_{_{\mathrm{H}_2}}}} \tag{9}$$

The velocity of the hydrogen molecules in thermal equilibrium with the tungsten shield can be found by substituting Eq. 7 into Eq. 9.

$$v_{_{\mathrm{H}_2}} = \sqrt{2\,C_\mathrm{p}\,(T_\mathrm{shield} - T_{_\mathrm{LH2}})} \;=\; 9693\,\mathrm{m/s} \tag{10}$$

The thrust due to the high velocity hydrogen molecules can be found by multiplying the mass flow rate $\dot{m}$ (Eq. 8) with the velocity of hydrogen molecules $v_{_{\mathrm{H}_2}}$ (Eq. 10)

$$F_{_{\mathrm{H}_2}} = hA\sqrt{\frac{2\,(T_\mathrm{shield} - T_{_\mathrm{LH2}})}{C_\mathrm{p}}} \;=\; 142.3\,A\,(\mathrm{newtons}) \tag{11}$$

where A is the area of the tungsten shield. The thrust/mass provided by evaporative cooling (Eq. 11) is more substantial than the thrust/mass produced by the *photon drive* described in the first half of this paper. The thrust/mass (in units of g's) produced by positron heating and subsequent LH2 evaporative cooling is

$$\frac{F_{_{\mathrm{H}_2}}}{m(g)} = \frac{142.3\,A}{A\rho t(g)} = 0.043\ \mathrm{g's} \tag{12}$$

compared to 1.50×10^{-6} g's for the *photon drive*. Therefore, the thrust due to the *photon drive* can be ignored in future calculations. Furthermore, the specific impulse due to evaporative cooling can be found by combining equations 8 and 11

$$(I_\mathrm{sp})_{_{\mathrm{H}_2}} = \frac{F}{\dot{m}_{_\mathrm{LH2}} g} = \frac{\sqrt{2\,C_\mathrm{p}\,(T_\mathrm{shield} - T_{_\mathrm{LH2}})}}{g} \;=\; 988\,\mathrm{s} \tag{13}$$

While this represents a significant decrease in the I_sp when compared to the *photon drive*, it is still more than a factor of 2 times higher than the most efficient chemical engines currently available (455 s at sea level). The engine, as described so far, is similar to the NERVA engines of the 1960's. However, instead of using uranium fission as the source of thermal energy, this concept uses positrons. An antimatter thermal rocket will have the same performance as a nuclear thermal rocket but will not require the large mass of a nuclear reactor.

One possible method to increase the thrust-to-weight ratio using the evaporative cooling technique is to slice the tungsten shield into ten separate layers with a combined thickness of 5 radiation lengths. This will allow the hydrogen gas to circulate between the layers and effectively increase the surface area (by approximately a factor of 20) over which the evaporative cooling can occur. For a tungsten shield with a radius of 10 m, the shield will have a surface area of 12,566 m^2, a mass of 212,215 kg, a convective energy transfer of 8,666 MW and a thrust of 1.79 MN.

However, the performance characteristics for a 10 m shield are unnecessarily large (mega-newtons). Once a spacecraft is launched, an engine with tens of kilo-newtons should be sufficient. As a result, a more practical design might be a shield with a radius of 1.5 m. The operating characteristics for such an engine would be a surface area of 282.6 m^2, a mass of 4,828 kg, a power of 194.8 kW, and a thrust of 40.2 kN.

B. Velocity boost due to evaporative cooling

The velocity boost due to evaporative cooling is determined from the familiar equation

$$\Delta v = v_{_{\mathrm{H}_2}}\, \ell n \left(\frac{m_{\mathrm{initial}}}{m_{\mathrm{final}}} \right) \tag{14}$$

where $v_{_{\mathrm{H}_2}}$ is described in equation 10.

The amount of LH2 required to obtain a specific Δv as a function of the inner surface area of the shield for a 30,000 kg payload is shown in Fig. 3. The function displayed in this figure is described by the following equation

$$\Delta v = v_{_{\mathrm{H}_2}} \ell n \left(\frac{M_{\mathrm{initial}}}{M_{\mathrm{final}}} \right) = 9692.7(\mathrm{m/s})\ \ell n \left(\frac{\text{mass of hydrogen + mass of shield + 30,000 kg}}{\text{mass of shield + 30,000 kg}} \right) \tag{15}$$

The total mass of the system increases proportionately with the inner area of the shield. With an inner shield radius of 1.5 m, a Δv of 20 km/s can be obtained with 240,000 kg of liquid hydrogen. As the inner shield radius and area increase, the total mass of hydrogen required to obtain a Δv of 20 km/s obviously increases. The Δv range (0-20 km) used Fig. 3 includes the range of Δv's required for a Hohman transfer to all 8 planets in our solar system. The Δv values shown on the y-axis is the total change in velocity including Δv_{burn}, $\Delta v_{\mathrm{retro}}$, and $\Delta v_{\mathrm{plane\,change}}$ assuming the plane change occurs at a true anomaly of 90^o.

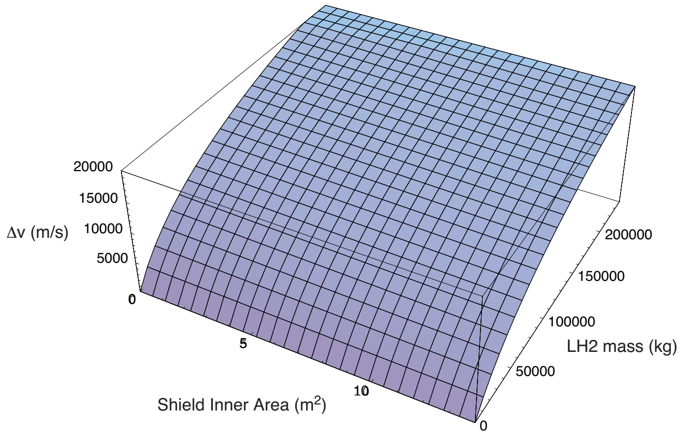

Figure 3. The Δv as a function of shield inner area and LH2 mass.

Finally, the mass of antimatter (i.e., positrons) required to obtain a specific Δv is shown in Fig. 4. The mass flow rate of positrons $\dot{m}$ is determined by the thermal power transferred from the shield to the LH2 and dividing it by the speed of light squared.

$$\dot{m} = \dot{Q}/c^2 \tag{16}$$

This can be multiplied by the burn time to determine the total mass of positrons required for the mission. The burn time can be found by taking the mass of the LH2 propellant required (Eq. 15) and dividing it by the mass flow rate $\dot{m}_{_{\mathrm{LH2}}}$ of the LH2 (Eq. 8).

IV. Conclusions

Relying on the photon momentum alone for the engine thrust (i.e., the *photon drive*) produces a relatively small thrust/mass ratio $\sim 1.5 \times 10^{-6}$ g's. On the other hand, the antimatter annihilation into gamma rays provide a very efficient heating source for producing a substantial thrust/mass ratio ≥ 0.043 g's. The photon absorption on a tungsten shield appears to give sustainable thermal energies for heating liquid hydrogen as a propellant while only using a fraction of a gram of antimatter. If the containment of positrons can be safely confined in a small trap, only modest amounts ($\sim$ one milligram) of antimatter would be required for a single mission.

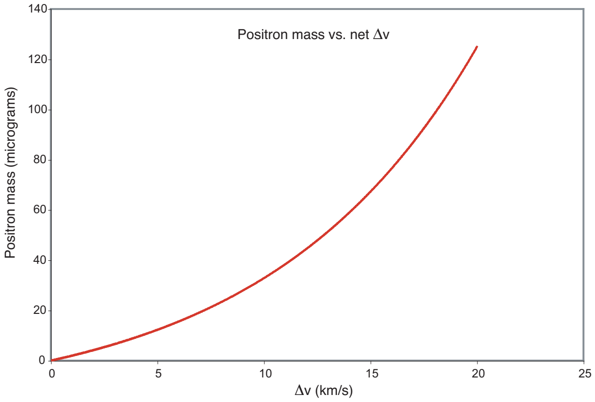

Figure 4. The mass of positrons required as a function Δv for a tungsten shield having a radius of 1.5 m and operating temperature of 3300 K.

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