

Chapter 8

Momentum and Impulse

Momentum plays a pivotal role in extending our understanding of Newton's Laws. In fact, Newton's laws were first written in terms of momentum. Later in this chapter, we will discover that, like energy, momentum is also a conserved quantity in our universe.

1 Momentum and Impulse

When we previously studied Newton's 2nd law, we wrote that:

$$\sum \vec{F} = m\vec{a}$$

However, if we write it in the following way (similar to what Newton did), we find:

$$\sum \vec{F} = m \frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v})$$

thus, the sum of the forces is equal to the time rate of $m\vec{v}$, which we call the **momentum**, or **linear momentum**.

$$\vec{p} = m\vec{v} \quad (\text{definition of momentum}) \quad (1)$$

Momentum (\vec{p}) is a vector quantity with SI units of kg·m/s.

If we write Newton's second law in terms of momentum, we find the following:

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \quad (2)$$

The advantage to writing Newton second law this way is that the $\sum \vec{F}$ is related to both the change in velocity vector (\vec{v}) and the change in mass m .

Using our definition of momentum Eq. 1, we decompose this vector equation into three scalar equations:

$$p_x = m v_x \quad p_y = m v_y \quad p_z = m v_z \quad (3)$$

It is more instructive to look at Eq. 1 when applying Newton's second law to a system where you want to investigate the motion of an object in all three dimensions at the same time. However, in solving most problems, we will more frequently use the scalar definitions described in Eq. 3. This is especially true once we've established the principle of *conservation of momentum*.

Ex. 3 Show that the kinetic energy K and the momentum magnitude p of a particle with mass m are related by $K = p^2/2m$. b) A 0.040-kg cardinal (*Richmondena cardinalis*) and a 0.145-kg baseball have the same kinetic energy. Which has the greater magnitude of momentum? What is the ratio of the cardinal's magnitude of momentum to the baseball's? c) A 700-N man and a 450-N woman have the same momentum. Who has the greater kinetic energy? What is the ratio of the man's kinetic energy to that of the woman?

Let's consider a particle that is acted upon by a *constant* net force $\sum \vec{\mathbf{F}}$ during a time interval Δt . We define the impulse to be the product of the net force and the time interval:

$$\vec{\mathbf{J}} = \sum \vec{\mathbf{F}} (t_2 - t_1) = \sum \vec{\mathbf{F}} \Delta t \quad (\text{assuming constant net force}) \quad (4)$$

The direction of the impulse is in the same direction as $\sum \vec{\mathbf{F}}$, and has SI units of kg·m/s, the same as linear momentum. In fact, this suggests that there might be a connection between *impulse* and *linear momentum*. Let's rewrite Newton's second law in the following manner:

$$\sum \vec{\mathbf{F}} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t} = \frac{\vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1}{t_2 - t_1}$$

Then, rearranging terms we have:

$$\sum \vec{\mathbf{F}} (t_2 - t_1) = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1$$

Using our definition of *impulse* from Eq. 4, we arrive at the **impulse-momentum theorem**:

$$\vec{\mathbf{J}} = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 \quad (\text{impulse-momentum theorem}) \quad (5)$$

The change in momentum of a particle equals the net force multiplied by the time interval over which the net force is applied.

If the $\sum \vec{\mathbf{F}}$ is not constant, we can integrate both sides of Newton's second law $\sum \vec{\mathbf{F}} = d\vec{\mathbf{p}}/dt$ over the time interval $t_1 \rightarrow t_2$:

$$\int_{t_1}^{t_2} \sum \vec{\mathbf{F}} dt = \int_{t_1}^{t_2} \frac{d\vec{\mathbf{p}}}{dt} dt = \int_{\vec{\mathbf{p}}_1}^{\vec{\mathbf{p}}_2} d\vec{\mathbf{p}} = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1$$

where the integral on the left-hand side is defined to be the impulse $\vec{\mathbf{J}}$ due to the net force $\sum \vec{\mathbf{F}}$ over the time interval $(t_1 \rightarrow t_2)$:

$$\vec{\mathbf{J}} = \int_{t_1}^{t_2} \sum \vec{\mathbf{F}} dt \quad (\text{general definition of impulse})$$

We can define an *average* net force $\vec{\mathbf{F}}_{\text{av}}$ such that even when $\sum \vec{\mathbf{F}}$ is not constant, the impulse $\vec{\mathbf{J}}$ is given by

$$\vec{\mathbf{J}} = \vec{\mathbf{F}}_{\text{av}} (t_2 - t_1) \quad (6)$$

Ex. 8 Force of a Baseball Swing. A baseball has a mass 0.145 kg. a) If the velocity of a pitched ball has a magnitude of 45.0 m/s and the batted ball's velocity is 55.0 m/s in the opposite direction, find the magnitude of the change in momentum of the ball and of the impulse applied to it by the bat. b) If the ball remains in contact with the bat for 2.00 ms, find the magnitude of the average force applied by the bat.

1.1 Momentum and Kinetic Energy Compared

When a net force $\sum \vec{\mathbf{F}}$ is applied to a system, the kinetic energy and the momentum undergo change according to the following equations:

$$\begin{aligned} \Delta \vec{\mathbf{p}} &= \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 = \left(\sum \vec{\mathbf{F}} \right) \Delta t \\ \Delta K &= K_2 - K_1 = \left(\sum \vec{\mathbf{F}} \right) \cdot \vec{\mathbf{s}} \end{aligned}$$

2 Conservation of Momentum

When two bodies A and B interact with each other but not with anything else, the forces exerted on each body must be described by Newton's *third* law, namely, the

two forces are always equal in magnitude and opposite in direction. Furthermore, the forces are internal forces, so the sum of the forces must be equal to zero.

$$\vec{\mathbf{F}}_{\text{B on A}} + \vec{\mathbf{F}}_{\text{A on B}} = \vec{\mathbf{0}}$$

$$\vec{\mathbf{F}}_{\text{B on A}} + \vec{\mathbf{F}}_{\text{A on B}} = \frac{d\vec{\mathbf{p}}_A}{dt} + \frac{d\vec{\mathbf{p}}_B}{dt} = \frac{d}{dt}(\vec{\mathbf{p}}_A + \vec{\mathbf{p}}_B) = \vec{\mathbf{0}} \quad (7)$$

Now we define the total momentum of the two-body system to be:

$$\vec{\mathbf{P}} = \vec{\mathbf{p}}_A + \vec{\mathbf{p}}_B \quad (\text{the total momentum})$$

Substituting our equation for *total momentum* into Eq. 7, we find:

$$\vec{\mathbf{F}}_{\text{B on A}} + \vec{\mathbf{F}}_{\text{A on B}} = \frac{d\vec{\mathbf{P}}}{dt} = \vec{\mathbf{0}}$$

If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.

This is the **principle of conservation of momentum**. The total momentum can be expanding to N particles:

$$\vec{\mathbf{P}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \vec{\mathbf{p}}_3 + \cdots = \sum_{i=1}^N \vec{\mathbf{p}}_i \quad (\text{the total momentum})$$

Again, this is a vector equation that really represents three scalar equations:

$$P_x = p_{1x} + p_{2x} + p_{3x} + \cdots = m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} + \cdots$$

$$P_y = p_{1y} + p_{2y} + p_{3y} + \cdots = m_1 v_{1y} + m_2 v_{2y} + m_3 v_{3y} + \cdots$$

$$P_z = p_{1z} + p_{2z} + p_{3z} + \cdots = m_1 v_{1z} + m_2 v_{2z} + m_3 v_{3z} + \cdots$$

N.B. Momentum is separately conserved in the x , y , and z directions.

Ex. 14 A 68.5-kg astronaut is doing a repair in space on the orbiting space station. She throws a 2.25-kg tool away from her at 3.20 m/s relative to the space station. With what speed and in what direction will she begin to move?

Ex. 23 The nucleus of ^{214}Po decays radioactively by emitting an alpha particle (mass 6.65×10^{-27} kg) with kinetic energy 1.23×10^{-12} J, as measured in the laboratory reference frame. Assuming that the Po was initially at rest in this frame, find the recoil velocity of the nucleus that remains after the decay.

3 Inelastic Collisions

First of all, we need to define what constitutes a collision. When we think of collisions, we naturally conjure up a picture of two objects coming into contact with each other, possibly doing some damage to each other, and possibly fragmenting off bits and pieces in the process. However, this describes only one class of collisions, *inelastic* collisions. There is another class of collisions called *elastic* collisions where physical contact may or may not occur. We will describe elastic collisions in the next section.

Inelastic collisions come in two types: inelastic, and *completely* inelastic collisions. If two objects collide and “stick” together and move off together as “one” object, then this is called an **inelastic collision**.

Let’s look at a collision between two particle A and B whose initial and final motions are confined to the x direction. Let’s label the initial velocities with the subscript 1 and the final velocities with the subscript 2. **In an inelastic collision, momentum is the only quantity conserved.**

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x} \quad (\text{for all collisions})$$

$$m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B) v_f \quad (\text{for a completely inelastic collision})$$

Ex. 30 Two fun-loving otters are sliding toward each other on a muddy (and hence frictionless) horizontal surface. One of them, of mass 7.50 kg, is sliding to the left at 5.00 m/s, while the other, of mass 5.75 kg, is slipping to the right at 6.00 m/s. They hold fast to each other after they collide. (a) Find the magnitude and direction of the velocity of these free-spirited otters right after they collide. (b) How much mechanical energy dissipates during this play?

Ex. 39 A Ballistic Pendulum. A 12.0-g rifle bullet is fired with a speed of 380 m/s into a ballistic pendulum with mass 6.00 kg, suspended from a cord 70.0 cm long (see Example 8.8 in Section 8.3). Compute a) the vertical height through which the pendulum rises; b) the initial kinetic energy of the bullet; c) the kinetic energy of the bullet and pendulum immediately after the bullet becomes embedded in the pendulum.

4 Elastic Collisions

In an **elastic collision**, the momentum and kinetic energy are both conserved.

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x} \quad (\text{for all collisions})$$

$$\frac{1}{2}m_A v_{A1x}^2 + \frac{1}{2}m_B v_{B1x}^2 = \frac{1}{2}m_A v_{A2x}^2 + \frac{1}{2}m_B v_{B2x}^2 \quad (\text{Conservation of KE})$$

Let's imagine we have a collision in one dimension (x -axis) between objects A and B , and the resulting motion after the collision is also along the x axis. We will let the subscript "i" denote the velocities before the collision and the subscript "f" denote the velocities after the collision. In this situation, we have both *conservation of kinetic energy* and *conservation of momentum*. If we know the initial velocities, we can find the final velocities by applying these two conservation laws:

In one dimension:

$$v_{Af} = \frac{m_A - m_B}{m_A + m_B} v_{Ai} + \frac{2m_B}{m_A + m_B} v_{Bi}$$

$$v_{Bf} = \frac{2m_A}{m_A + m_B} v_{Ai} + \frac{m_B - m_A}{m_A + m_B} v_{Bi}$$

Ex. 46 You are at the controls of a particle accelerator, sending a beam of 1.50×10^7 m/s protons (mass m) at a gas target of an unknown element. Your detector tells you that some protons bounce straight back after a collision with one of the nuclei of the unknown element. All such protons rebound with a speed of 1.20×10^7 m/s. Assume that the initial speed of the target nucleus is negligible and that the collision is elastic. a) Find the mass of one nucleus of the unknown element. Express your answer in terms of the proton mass m . b) What is the speed of the unknown nucleus immediately after such a collision?

5 Center of Mass

$$x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$y_{\text{cm}} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$z_{\text{cm}} = \frac{m_1z_1 + m_2z_2 + m_3z_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

Ex. 48 Find the position of the center of mass of the system of the sun and Jupiter. (Since Jupiter is more massive than the rest of the planets put together, this is essentially the position of the center of mass of the solar system.) Does the center of mass lie inside or outside the sun? Use the data in Appendix F. ($M_{\text{Sun}} = 1.99 \times 10^{30}$ kg, and $M_{\text{Jupiter}} = 1.90 \times 10^{27}$ kg)

5.1 Motion of the Center of Mass

$$V_{\text{cm-x}} = \frac{m_1v_{1x} + m_2v_{2x} + m_3v_{3x} + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$V_{\text{cm-y}} = \frac{m_1v_{1y} + m_2v_{2y} + m_3v_{3y} + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$V_{\text{cm-z}} = \frac{m_1v_{1z} + m_2v_{2z} + m_3v_{3z} + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$\vec{\mathbf{V}}_{\text{cm}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + m_3\vec{\mathbf{v}}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\vec{\mathbf{P}}}{M} \quad (8)$$

The total momentum can be written as:

$$\vec{\mathbf{P}} = M \vec{\mathbf{V}}_{\text{cm}}$$

5.2 External Forces and Center-of-Mass Motion

If the net external force on a system of particles is not zero, then the total momentum is not conserved and the velocity of the center-of-mass (\vec{v}_{cm}) changes. Because of Newton's third law, the internal forces all cancel in pairs, and $\sum \vec{F}_{int} = \vec{0}$. What survives on the left-hand side of Newton's second law is the sum of the *external* forces:

$$\sum \vec{F}_{ext} = M\vec{a} \quad (\text{where } M \text{ is the mass of the body or collection of particles})$$

When a body of a collection of particles is acted on by external forces, the center-of-mass moves just as though all the mass were concentrated at that point and it were acted on by a net force equal to the sum of the external forces on the system.

$$\sum \vec{F}_{ext} = M\vec{a}_{cm} = \frac{d\vec{P}}{dt}$$

Note:

If the net external force is zero, then the acceleration of the center-of-mass is zero (i.e., $\vec{a}_{cm} = 0$). Likewise, if the net external force is zero, then the total momentum \vec{P} must be *conserved*, (i.e., $d\vec{P}/dt = 0$).

6 Rocket Propulsion

Using conservation of momentum, we can write the following:

The Thrust

$$F = -v_{\text{ex}} \frac{dm}{dt}$$

The Velocity

$$v - v_o = v_{\text{ex}} \ln \left(\frac{m_o}{m} \right)$$

- Ex. 57** A 70-kg astronaut floating in space in a 110-kg MMU (manned maneuvering unit) experiences an acceleration of 0.029 m/s^2 when he fires one of the MMU's thrusters. a) If the speed of the escaping N_2 gas relative to the astronaut is 490 m/s , how much gas is used by the thruster in 5.0 s ? b) What is the thrust of the thruster?
- Ex. 85** A neutron with mass m makes a head-on, elastic collision with a nucleus of mass M , which is initially at rest. a) Show that if the neutron's initial kinetic energy is K_o , the kinetic energy that it loses during the collision is $4mMK_o/(M+m)^2$. b) For what value of M does the incident neutron lose the most energy? c) When M has the value calculated in part (b), what is the speed of the neutron after the collision?

Homework – Chapter 8

Exercises: 3, 6, 8, 10, 14, 19, 23, 26, 30, 33, 35, 39, 46, 48, 50, 51, 57

Problems: 85, 93, 95