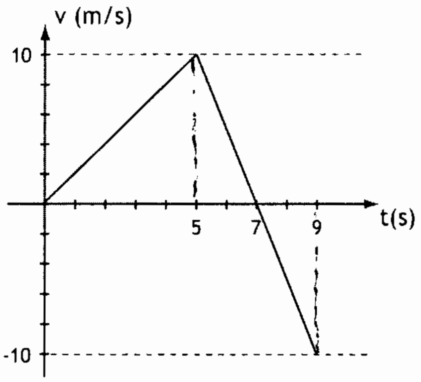
Final Exam PS150
April 25, 2009

Answer Key
Name

Show your work !!

10 points



1. The velocity of an object moving in one dimension (the x-direction) is described be the velocity profile in the figure to the right.

   a. What is the final displacement after 9.00 sec?

Area from 0-5 sec. $= \frac{1}{2}\ 10\ m/s\ (5s) = 25\ m$

$x_f =$ 25 meters

   b. What is the maximum displacement during the motion from 0.00 to 9.00 sec?

Area from 0-7 sec $= \frac{1}{2}\ 10\ m\ (7s) = 35\ m$

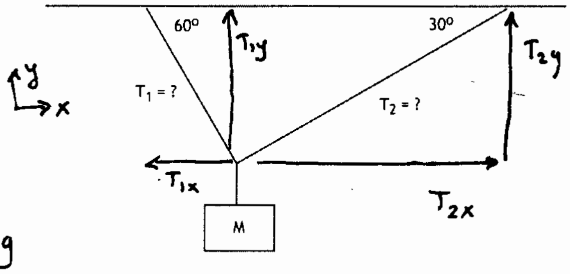
$x_{max} =$ 35 meters

   c. What is the acceleration at t = 7.00 sec?

Slope $= \frac{\Delta v}{\Delta t} = \frac{(-10-10)\ m/s}{4\ s} = -5\ m/s^2$

$a =$ -5.00 m/s^2

10 points



2. A 3.00-kg object hangs motionless while suspended by three massless cords as show in the figure to the right.

   a. Calculate the tension T_1.

$\Sigma F_x = 0 \quad T_{2x} - T_{1x} = 0 \Rightarrow T_2\cos 30° - T_1\cos 60° = 0$

$\Sigma F_y = 0 \quad T_{1y} + T_{2y} - Mg = 0 \Rightarrow T_1\sin 60° + T_2\sin 30° = Mg$

$\boxed{T_1 = T_2\frac{\cos 30°}{\cos 60°}}$ or $\boxed{T_2 = T_1\frac{\cos 60°}{\cos 30°}}$

$T_1 =$ 25.46 N

   b. Calculate the tension T_2.

$T_1\sin 60° + \left(T_1\frac{\cos 60°}{\cos 30°}\right)\sin 30° = Mg$

$T_1\left(\sin 60° + \cos 60°\tan 30°\right) = Mg \qquad T_1 = \frac{Mg}{\sin 60° + \cos 60°\tan 30°}$

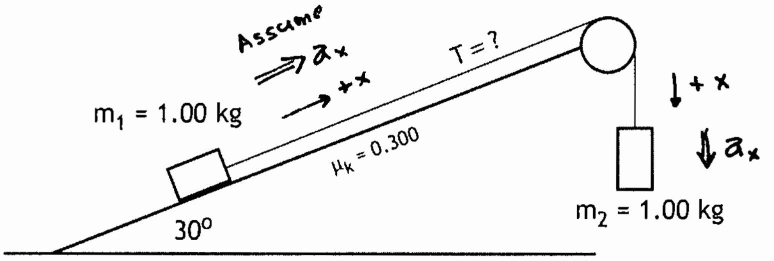
$T_2 =$ 14.7 N

$T_1 = \frac{3.00\ kg\ (9.8\ m/s^2)}{\sin 60° + \cos 60°\tan 30°} = \underline{\underline{25.46\ N}}$

$T_2 = T_1\frac{\cos 60°}{\cos 30°} = 25.46\ N\left(\frac{\cos 60°}{\cos 30°}\right) = \underline{\underline{14.70\ N}}$

10 points

3. Two masses are connected by a massless string that passes over a frictionless (massless) pulley. If the coefficient of kinetic friction on the incline is 0.300,



a. Find the acceleration of the two masses.

$$\Sigma F_x = (m_1+m_2)\,a_x \;\Rightarrow\; -m_1 g \sin 30° - \mu_k F_N + m_2 g = (m_1+m_2)\,a_x$$

$$-m_1 g \sin 30° - \mu_k\, m g \cos 30° + m_2 g = (m_1+m_2)\,a_x \;\Rightarrow\; a_x = \frac{-m_1 g\,(\sin 30° + \mu_k \cos 30°) + m_2 g}{(m_1+m_2)}$$

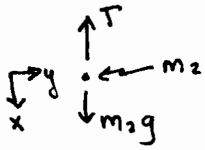
$$a_x = \frac{-1.0\,kg\,(9.8\,m/s^2)\left(\tfrac{1}{2} + 0.300\,\sqrt{3}/2\right) + 1.0\,kg\,(9.8\,m/s^2)}{2.00\,kg} = 1.18\,m/s^2$$

$a =$ 1.18 m/s^2

Good !!

We assumed the direction of the acceleration correctly.

b. Find the tension T in the massless string.



$$\Sigma F_x = m_2 a_x \;\Rightarrow\; m_2 g - T = m_2 a_x$$

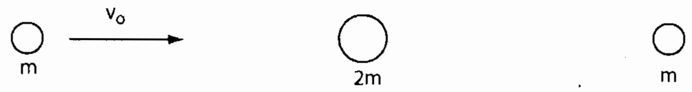
$$T = m_2 (g - a_x) = 1.0\,kg\,(9.8 - 1.18)\,m/s^2$$

$$T = 8.62\,N$$

$T =$ 8.62 N

10 points

4. Three titanium spheres form a straight line along the x axis. The first sphere is traveling with a velocity v_o while the 2nd and 3rd spheres are initially at rest. After all the collisions have occurred, find the final velocities of the spheres. Assume all the collisions are elastic and along the x-direction.



1st collision

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_o + 0 \Rightarrow V_{1f} = \frac{-m}{3m} V_o = -\frac{1}{3} V_o$$

$V_{1f} =$ −1/3 v_o

$$V_{2f} = \frac{2m_1}{m_1+m_2} + 0 \Rightarrow V_{2f} = \frac{2m}{3m} V_o = \frac{2}{3} V_o$$

2nd collision: $V_{2f} \rightarrow V_{2i} = \frac{2}{3} V_o$

$$V_{2f} = \frac{m_2 - m_3}{m_2 + m_3} V_{2i} + 0 \Rightarrow \frac{m}{3m}\,\frac{2}{3} V_o = \frac{2}{9} V_o$$

$V_{2f} =$ 2/9 v_o

$$V_{3f} = \frac{2(2m)}{3m} V_{2i} + 0 \Rightarrow \frac{4}{3}\,\frac{2}{3} V_o = \frac{8}{9} V_o$$

$V_{3f} =$ 8/9 v_o

b. Find the total kinetic energy of the spheres after all the collisions have occurred?

$$\text{Total } KE = \tfrac{1}{2} m_1 V_{1f}^2 + \tfrac{1}{2} m_2 V_{2f}^2 + \tfrac{1}{2} m_3 V_{3f}^2$$

$$KE_{TOTAL} = \tfrac{1}{2} m \left(-\tfrac{1}{3} V_o\right)^2 + \tfrac{1}{2}(2m)\left(\tfrac{2}{9} V_o\right)^2 + \tfrac{1}{2} m \left(8/9\, V_o\right)^2$$

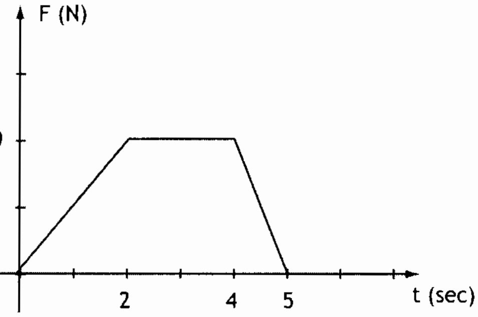
$$KE_{TOTAL} = \tfrac{1}{2} m \left(\tfrac{1}{9} + \tfrac{8}{81} + \tfrac{64}{81}\right) V_o^2 = \tfrac{1}{2} m V_o^2 \left(\tfrac{81}{81}\right) = \tfrac{1}{2} m V_o^2$$

$KE_{total} =$ 1/2 mv_o^2

As it should be... because $KE_{initial} = \frac{1}{2} m V_o^2$!!

10 points

5. An external force is applied to a 2.00-kg mass according the force diagram at the right. If the mass is originally at rest at t= 0.00 sec, what is its velocity after 5.00 seconds?



$J = \text{impulse} = \int F\,dt = \text{Area of a trapezoid} = m\Delta v$

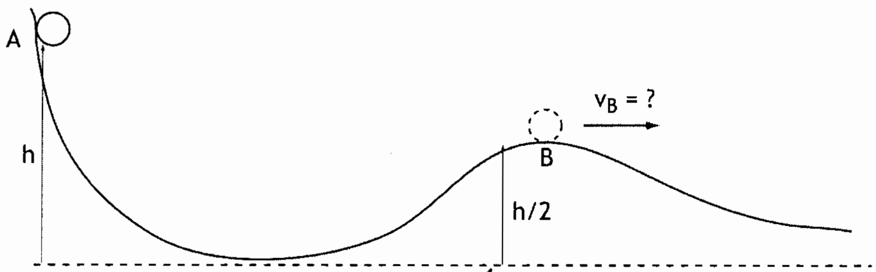
$J = \frac{(2+5)\text{sec}}{2} \times 200\text{N} = 700\ \text{N}\cdot\text{s} = 2.00\,\text{kg}\,(v_f - \underbrace{v_i}_{=0})$

$v_f = \frac{700\ \text{N}\cdot\text{s}}{2.0\,\text{kg}} = 350\ \text{m/s}$

$\boxed{v_f = 350\ \text{m/s}}$

10 points

6. A homogenous sphere of mass M is released from rest (point A) at a height h =2.00 m above the ground as shown in the figure below. The mass of the sphere is 2.00 kg and its radius is 10.0 cm. Assuming that it's moment of inertia is $\frac{2}{5}MR^2$, find its velocity at point B. ***Assume that the sphere rolls without slipping.***



$W_{grav} = \Delta K_{trans.} + \Delta K_{rot.} = \frac{1}{2}mv_f^2 - \cancel{\frac{1}{2}mv_i^2} + \frac{1}{2}I\omega_f^2 - \cancel{\frac{1}{2}I\omega_i^2} = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{v_f^2}{R^2}$

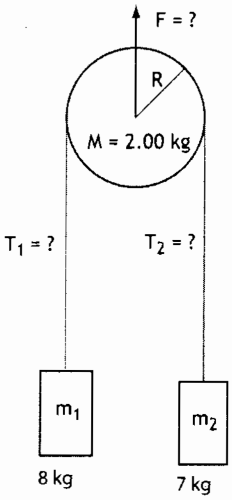
$W_{grav} = mg\frac{h}{2} = \frac{1}{2}mv_f^2 + \frac{1}{5}mv_f^2 \Rightarrow mg\frac{h}{2} = \frac{7}{10}mv_f^2 \Rightarrow \boxed{v_f^2 = \frac{5}{7}gh}$

$v_f = \sqrt{\frac{5}{7}(9.8\,\text{m/s}^2)(2.00\,\text{m})} = \underline{3.74\ \text{m/s}}$

v_B = 3.74 m/s

20 points

7. Two masses are connected by a massless string passing over a pulley of mass M and radius R as shown in the figure to the right. The string moves over the pulley without slipping causing the pulley to turn. Assume the axle of the pulley is frictionless (and motionless). The masses are released and free to move. Assume $I_{cm}^{disk} = \frac{1}{2}MR^2$ with a mass M = 2.00 kg, and a radius R = 0.200 m.



a. Find the acceleration of the two-mass system.

$+y \downarrow$ $\uparrow +y$

m_1: $\Sigma F_y = m_1 a_y \Rightarrow m_1 g - T_1 = m_1 a_y$ $\boxed{T_1 = m_1(g - a_y)}$ ①

m_2: $\Sigma F_y = m_2 a_y \Rightarrow T_2 - m_2 g = m_2 a_y$ $\boxed{T_2 = m_2(g + a_y)}$ ②

M: $\Sigma \tau = I\alpha \Rightarrow T_1 R - T_2 R = \frac{1}{2}MR^2\frac{a_y}{R}$ $\boxed{T_1 - T_2 = \frac{1}{2}Ma_y}$ ③

Substitute ① & ② into ③:

$m_1 g - m_1 a_y - m_2 g - m_2 a_y = \frac{1}{2}Ma_y$

$m_1 g - m_2 g = a_y\left(\frac{1}{2}M + m_1 + m_2\right)$

$a_y = \frac{m_1 g - m_2 g}{\frac{1}{2}M + m_1 + m_2}$

$a_y = \frac{(8-7)9.8\ \text{m/s}^2}{1\,\text{kg} + 8\,\text{kg} + 7\,\text{kg}} = 0.61$

a = 0.61 m/s²

b. Find the tension T_1.

① $T_1 = m_1(g - a_y) = 8\,kg(9.8 - 0.61)\,m/s^2 = 73.50\ N$

$T_1 =$ 73.5 N

c. Find the tension T_2.

$T_2 = m_2(g + a_y) = 7\,kg(9.8 + 0.61)\,m/s^2 = 72.89$

$T_2 =$ 72.9 N

d. Find F, the force required to keep the pulley from accelerating upward or downward.

$\Sigma F_y = 0 \Rightarrow F - T_1 - T_2 - Mg = 0 \Rightarrow F = Mg + T_1 + T_2$

$F = 2\,kg(9.8\,m/s^2) + (73.5 + 72.9)\,N = 166\ N$

$F =$ 166 N

20 points

8. The flywheel on your car is initially moving with angular velocity of 4,000 rpms. You decide to change to the next gear and the clutch plate (as well as other sources of friction) slow the flywheel down to a final angular velocity of 2,000 rpms (rev/min). This is accomplished in 2.00 seconds.

a. Find the angular deceleration. ***Assume that the deceleration is constant.***

$\omega_i = 4{,}000\,\frac{rev}{min}\left(\frac{2\pi\,rad}{rev}\right)\left(\frac{1\,min}{60s}\right) = 418.9\ rad/s \qquad \omega_f = 2{,}000\,\frac{rev}{min}\left(\frac{2\pi\,rad}{rev}\right)\left(\frac{1\,min}{60s}\right) = 209.4\ rad/s$

$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(209.4 - 418.9)\,rad/s}{2.00\ s} = -104.7\ rad/s^2$

$\alpha =$ -104.7 rad/s²

b. If we assume the flywheel is a flat disk of 25.0-kg mass and 20.0-cm radius, what is the average torque applied to the flywheel while it is decelerating? $I_{cm}^{disk} = \frac{1}{2}MR^2$

$\tau = I\alpha = \frac{1}{2}MR^2\alpha = \frac{1}{2}(25.0\,kg)(0.20m)^2(104.7\,rad/s^2) = \underline{\underline{52.36\ N\cdot m}}$

$\tau =$ 52.4 N·m

c. How much work is required to reduce the speed of the disk from 4,000 rpms to 2,000 rpms?

$W = \tau\,\Delta\theta$ or $W = \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = \frac{1}{2}I(\omega_f^2 - \omega_i^2)$

$W = \Delta K = \frac{1}{2}\left(\frac{1}{2}MR^2\right)(\omega_f^2 - \omega_i^2) = \frac{1}{4}(25.0kg)(0.20m)^2(209.4^2 - 418.9^2)\,rad^2/s^2$

$W = -32{,}899\ J = -3.29 \times 10^4\ J \qquad \boxed{W = -3.29 \times 10^4\ J}$

Work = -3.29×10^4 J

d. What is the average power dissipated from the flywheel as it slows down from 4,000 rpms to 2,000 rpms?

$Power = \frac{Work}{time} = \frac{-3.29 \times 10^4\ J}{2.sec} = -1.64 \times 10^4\ W$

$= -16.4\ kW$

$Power_{avg} =$ 1.64×10^4 watts

5 points (extra credit)

e. What is the change in angular momentum of the disk when it decelerates from 4,000 rpms to 2,000 rpms?

$\tau = \frac{\Delta L}{\Delta t} \qquad \Delta L = \tau\,\Delta t = 52.4\ N\cdot m\,(2\ s)$

$\Delta L = 104.8\ kg\cdot m^2/s$

or $\Delta L = I(\Delta\omega) = I(\omega_f - \omega_i)$

$\Delta L =$ 104.8 kg·m²/s