Exam #1 PS150
February 17, 2009

Show your work!

Answer Key
Name

15 points

1. Using the following three vectors:

$\vec{A} = \hat{i} - 2\hat{j}$

$\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{C} = \hat{i} + \hat{j} + ?\hat{k}$

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 0 \\ 2 & 1 & -1 \end{vmatrix} = \hat{i}(2-0) + \hat{j}(0+1) + \hat{k}(1+4)$

a.) Calculate $\vec{A} \times \vec{B}$.

$\vec{A} \times \vec{B} = \underline{2}\,\hat{i} + \underline{1}\,\hat{j} + \underline{5}\,\hat{k}$

b.) Calculate the magnitude of $\vec{A} \times \vec{B}$.

$|\vec{A} \times \vec{B}| = \sqrt{2^2 + 1^2 + 5^2} = \sqrt{30}$

$|\vec{A} \times \vec{B}| = \underline{\sqrt{30}}$

c.) What is the third component of $\vec{C}$ such that $\vec{B}$ is perpendicular to $\vec{C}$?

$\vec{B} \cdot \vec{C} = 0 \Rightarrow (2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} + \hat{j} + C_z \hat{k}) = 2 + 1 - C_z = 0$

$C_z = \underline{3}$

10 points

2. A spherical object is released from rest 100 meters above the ground. *Ignoring air resistance,*

a.) How far will the object fall before reaching a velocity of 27 m/s (~60 mph)?

$v_{oy} = 0$
$v_y = -27\ m/s$
$a_y = -9.8\ m/s^2$
$y = ?$
$t = ?$

$v_y^2 = v_{oy}^2 + 2a_y y \qquad y = \dfrac{v_y^2 - v_{oy}^2}{2a_y}$

$y = \dfrac{(-27)^2 - 0^2}{2(-9.8)} = -37.2 \text{ meters}$

distance = $\underline{37.2}$ *m*

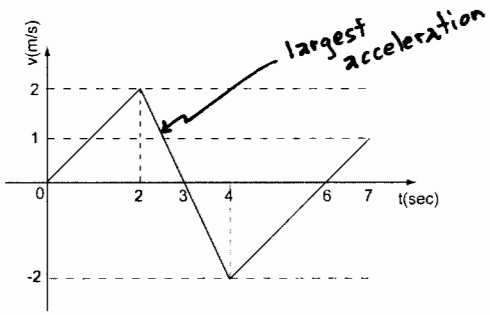
b.) How much time will it take for the object to reach this velocity (27 m/s)?

$v_y = v_{oy} + a_y t \qquad t = \dfrac{v_y - v_{oy}}{a_y} = \dfrac{-27\ m/s - 0\ m/s}{-9.8\ m/s^2} = 2.755$

$t = \underline{2.76}$ *sec.*

10 points

3. Using the following *v-t* diagram, calculate the following:



a.) What is the largest acceleration (deceleration) occurring during the 7-second interval (i.e., the largest acceleration in absolute magnitude)?

$a = \dfrac{\Delta v}{\Delta t} = \dfrac{-2-2}{2} = -2\ m/s^2$

$a_{max} = \underline{2}\ m/s^2$

deceleration

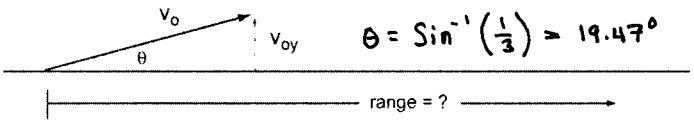
b.) Assuming that the object starts at the $x = 0\ m$ at $t = 0\ sec$, what is the net displacement after 7 seconds? Net displacement = areas under the triangles.

Area = $\frac{1}{2}(3\times2) - \frac{1}{2}(3\times2) + \frac{1}{2}(1\times1) = \frac{1}{2}$ meter

$x =$ 0.500 *meters*

15 points

4. A projectile is shot at an angle θ above level ground such that its muzzle velocity v_o is three times its vertical velocity component (i.e., $v_o = 3\ v_{oy}$). *Assuming no air resistance:*



a.) How high will the projectile travel if $v_o = 15$ m/s? $v_{oy}^2 = 2gh$

$h = \frac{v_{oy}^2}{2g} = \frac{\left(\frac{1}{3}\ 15\ m/s\right)^2}{2(9.8\ m/s^2)} = 1.28$ meters

max. height = 1.28 *m*

b.) What is the *range* of the projectile?

$R = \frac{v_o^2}{g}\sin 2\theta = \frac{(15\ m/s)^2}{(9.8\ m/s^2)}\sin(2\times 19.47^\circ) = 14.4$ meters

range = 14.4 *m*

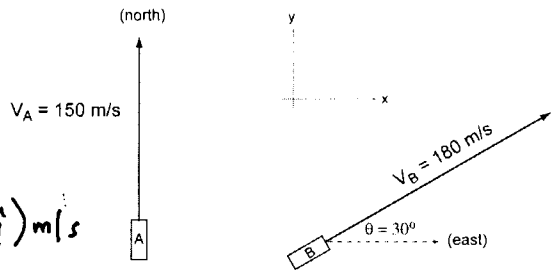
c.) At what other angle could the projectile be shot (without changing its muzzle velocity) and still have the same range?

$\theta_2 = 90^\circ - 19.47^\circ = 70.5^\circ$

θ_2 = 70.5° *degrees*

10 points

5. Two cars are traveling in straight lines on the Bonneville Salt Flats in Utah. Car *A* is traveling due north at 150 m/s while car *B* is traveling 30° *N* of *E* at 180 m/s. What is the velocity of car *A* with respect to car *B*?



$\vec{V}_{A/G} = (0\,\hat{i} + 150\,\hat{j})\ m/s$ $\quad \vec{V}_{B/G} = (155.9\,\hat{i} + 90\,\hat{j})\ m/s$

$\vec{V}_{A/B} = \vec{V}_{A/G} + \vec{V}_{G/B} = (0\,\hat{i} + 150\,\hat{j})\ m/s + (-155.9\,\hat{i} - 90\,\hat{j})\ m/s$

$\vec{V}_{A/B} = (-155.9\,\hat{i} + 60\,\hat{j})\ m/s$

$\vec{v}_{A/B} =$ (−155.9 $\hat{i}$ + 60.0 $\hat{j}$) m/s