Fluid Statics Chapter 15

Fluids and Solids

Solids - can normally be in equilibrium under applied compression, tensile, or shearing forces with only minimal changes in their size or shape.

Liquids - cannot produce reaction forces to applied forces in arbitrary directions. Most liquids are nearly incompressible, so they can provide reactions forces to compression forces with small changes in their volume.

Gases - cannot support compressional, tensile, or shearing forces. Compressional forces cause substantial changes in the state of the gas, and shearing forces also cause the molecules to flow in the direction of the force.

Fluids - Liquids and gases are classified as *fluids*. These materials will easily flow under the action of a shearing force.

Pressure and density

Fluids flow because they cannot sustain a force parallel to its surface. In this chapter we will study the static conditions (no flow), so we will focus on forces that are perpendicular to the surface of the fluid.

Independent of the shape of the fluid, forces at the fluid boundary are always perpendicular. The magnitude of the *force/area* is called the pressure

$$p = \frac{\Delta F}{\Delta A}$$

1 atmosphere $=$ 14.7 lbs/in^2 $=$ 1.01325×10^5 Pa (exactly)
1 Pa $=$ 1 N/m^2
1 bar $=$ 10^5 Pa (roughly 1 atmosphere)
1 torr $=$ 1 mm of mercury (Hg)

Examples of some pressures

Density

The density ρ of a small element of any material is the mass Δm of the element divided by its volume ΔV.

$$\rho = \frac{\Delta m}{\Delta V}$$

If the object is homogeneous, its density is constant and can be written as:

$$\rho = \frac{m}{V}$$

Examples of some densities

The Bulk Modulus

As the pressure increases, the volume decreases. The *bulk modulus* is defined as:

$$B = -\frac{\Delta p}{\Delta V / V}$$

Moduli, in general, are defined as:

$$\text{modulus} = \frac{\text{stress}}{\text{strain}} = \frac{\text{force/area}}{\text{fractional change in space}}$$

The following moduli are <u>not in this book</u>, but you should be aware of them.

Young's modulus

$$Y = \frac{F/A}{\Delta L/L_o}$$ measured when tensile or compression forces are present.

Shear modulus

$$S = \frac{F/A}{\Delta x/h}$$ measured when a force is applied parallel to the boundary.

Variation of pressure in a fluid at rest

When a fluid is in equilibrium, every portion of the fluid is in equilibrium. The *net force* and *net torque* must be zero. Consider a small disk of area A and thickness dy. How does the pressure change in the y direction.

Using Newton's 2^{nd} Law:

$$\sum F_y = pA - (p + dp)A - mg = 0$$

$$\frac{dp}{dy} = -\rho g$$

$$p_2 - p_1 = -\rho g(y_2 - y_1)$$

In some cases we measure the pressure as a function of depth h in a fluid.

$$p = p_o + \rho g h$$

Exercise 12 A swimming pool has the dimensions 80 ft × 30 ft ×8.0 ft. *(a)* When it is filled with water, what is the force (due to the water alone) on the bottom? On the ends? On the sides? *(b)* If you are concerned with whether or not the concrete walls will collapse, is it appropriate to take the atmospheric pressure into account?

Variation of Pressure in the Atmosphere

$$\frac{dp}{dy} = -\rho g$$

$$\frac{\rho}{\rho_o} = \frac{p}{p_o}$$

$$\frac{dp}{p} = -\frac{g\rho_o}{p_o}\, dy$$

$$p = p_o e^{-h/a} \qquad \text{where} \qquad a = \frac{p_o}{g\rho_o} = 8.55 \text{ km}$$

Problem 7 *(a)* Show that Eq. 15-13, the variation of pressure with altitude in the atmosphere (temperature assumed to be uniform), can be written in terms of density ρ as

$$\rho \;=\; \rho_o\, e^{-h/a}$$

(b) Assume that the drag force D due to the air on an object moving at speed v is given by $D = CA\rho v^2$ where C is a constant, A is the frontal cross-sectional area of the object, and ρ is the local air density. Find the altitude at which the drag force on a rocket is a maximum if the rocket is launched vertically and moves with constant upward acceleration a_r.

Pascal's Principle and Archimedes' Principle

Pascal's Principle

Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.

If you increase the external pressure on a fluid at one location by an amount Δp, the same increase in pressure is experienced everywhere in the fluid.

$$p = p_{\text{ext}} + \rho g h$$

An increase in the external pressure results in a change of pressure

$$\Delta p \;=\; \Delta p_{\text{ext}} + \Delta(\rho g h) \qquad \rightarrow \qquad \Delta p \;=\; \Delta p_{\text{ext}}$$

In a hydraulic lever, a change in pressure at one end is communicated throughout the fluid.

$$p_1 \;=\; p_2 \qquad \rightarrow \qquad \frac{F_1}{A_1} \;=\; \frac{F_2}{A_2}$$

If F_2 represents the weight of a car (mg), how much force F_1 must you apply to a lever to lift it?

$$F_1 \;=\; mg\,\frac{A_1}{A_2} \qquad \rightarrow \qquad F_1 = mg\left(\frac{r_1}{r_2}\right)^2$$

Archimedes' Principle

A body wholly or partially immersed in a fluid is buoyed up by a force equal in magnitude to the weight of the fluid displaced by the body.

The buoyant force

$$F_B \;=\; \rho_{\text{fluid}}\, g\, V_{\text{displaced}}$$

Exercise 17* The tension in a string holding a solid block below the surface of a liquid (of density greater than the solid) is T_o when the containing vessel (Fig. 15-18) is at rest. Show that the tension T, when the vessel has an upward vertical acceleration a, is given by $T_o(1 + a/g)$.

Exercise 21 A tin can has a total volume of 1200 cm^3 and a mass of 130 g. How many grams of lead shot could it carry without sinking in water? The density of lead is 11.4 g/cm^3.

The Measurement of Pressure

Barometers

$$1 \text{ torr} \;=\; 1 \text{ mm of Hg} \;=\; 133.322 \text{ Pa}$$

The height of mercury in a barometer is measured to be:

$$p = p_o + \rho g h \qquad \text{but} \qquad p_o \;=\; 0 \qquad \text{so} \qquad h = \frac{p}{\rho\, g}$$

Guage pressure

$$p - p_o \;=\; \rho g h$$

Absolute Pressure

$$p \;==\; p_o + \rho g h$$

Exercise 32 Estimate the density of the red wine that Pascal used in his 14-m-long barometer. Assume that the wine filled the tube.