1. An ensemble of quantum-mechanical harmonic oscillators are initially in the ground state \( |0\rangle \). The oscillators are subject to a single perturbation described by the following Hamiltonian:

\[
H'(x, t) = V_0 e^{-\alpha^2 x^2} e^{-\gamma^2 t^2}
\]

where \( V_0, \alpha \) and \( \gamma \) are constants. Use the following definition for the transition probability amplitude:

\[
c_{mk}(t) = \frac{1}{i\hbar} \int_{-\infty}^{\infty} e^{i \omega_{km} t} \langle \psi_m | H'(x, t) | \psi_k \rangle \, dt
\]

a. Calculate the probability for finding a system in the 2nd excited state \( |2\rangle \). Write your answer in terms of \( V_0, \alpha, \beta, \gamma, \omega_{km}, \) and \( \hbar \), where

\[
\omega_{km} = \omega_{k \to m} = \frac{E_m - E_k}{\hbar} = (m - k)\omega_o
\]

where \( \omega_o \) is the fundamental oscillating frequency of the harmonic oscillator, and \( \beta = \frac{m \omega_o}{\sqrt{\hbar}} \) is the constant used in the harmonic oscillator wave functions.

Answer: __________________________

b. Calculate the probability for finding a system in the 3rd excited state \( |3\rangle \).

Very little calculation is required to answer this question, but clearly state how you came to your answer.

Answer: __________________________
c. What value of $\gamma$ will result in the largest probability in part (a)? If you use Mathematica to find $\gamma$ at the maximum probability, remove the Abs[] (and other like functions) before using the Solve command. Write $\gamma$ in terms of $\omega_{km}$.

$$\gamma = \quad \text{___________}$$

d. Calculate the probability for finding a harmonic oscillator in the 2\textsuperscript{nd} excited state $|2\rangle$ after this single perturbation has occurred. Assume $V_0 = \frac{1}{2} \hbar \omega_0$ and $\alpha = 2\beta$.

Probability = \text{___________} \% 

20 points

2. A 3 MeV beam of $\alpha$ particles strikes an aluminum target.

a. Determine the distance of closest approach $D$ between the $\alpha$ particles and the aluminum nuclei at this energy. (note: this is the “head on” distance of closest approach).

$$D = \quad \text{___________} \text{ fm}$$

b. Calculate the number of aluminum nuclei per unit volume in the target. (Aluminum has $Z = 13$, $A = 27$, and density 2.70 g/cm\textsuperscript{3}, $m = 26.98$ g/mol)

$$\# \text{ Al nuclei / m}^3 = \quad \text{___________}$$
c. Suppose the beam of $\alpha$ particles has a flux of $10^5 \alpha$ particles/sec. If the thickness of the aluminum target is $10^{-4}$ cm, calculate the number of $\alpha$ particles scattered per second into the backward hemisphere.

Flux into the backward hemisphere = ________________ $\alpha$ particles/sec

d. What is the distance of closest approach for an $\alpha$ particle scattered at 90°?

$r_{min} = ________________$ fm

10 points

3. Calculate the energy released when $^4Be$ decays into two $\alpha$ particles.

$Q = _______ \text{ MeV}$

Calculate the energy released in the reaction

$$\frac{3}{2}H + \frac{3}{2}He \rightarrow \frac{6}{3}Li + \gamma$$

Include the energy of the photon in your final answer.

$Q = _______ \text{ MeV}$
10 points

4. Consider nuclei with small nucleon number $A$ such that $Z = N = A/2$. Neglecting the pairing term, and approximating $Z(Z-1) \approx Z^2$, show that the semi-empirical mass formula then gives the binding energy per nucleon to be

$$\frac{B}{A} = a_v - a_s A^{-1/3} - \frac{a_c}{4} A^{2/3}$$

Show that this expression reaches a maximum for $Z = A/2 = 26$ (iron).