Final Exam PS250
June 24, 2016

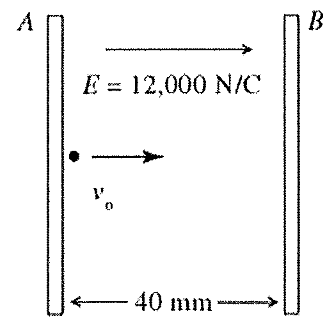
Show your work !!

Answer Key
Name

Incorrect Answers: You can earn partial credit if your work supports your answer.
Correct Answers: Points may be deducted if your work does not support your answer.

10 points

1. A pair of charged conducting plates produces a uniform field of 12,000 N/C, directed to the right between the plates. The separation of the plates is 40 mm. An electron is projected from plate *A* , directly toward plate *B*, with an initial velocity of $v_o = 2.0 \times 10^7$ m/s, as shown in the figure. The velocity of the electron as it strikes plate *B* is closest to: **(circle the correct answer)**
$m_e = 9.11 \times 10^{-31}$ kg



$$\frac{1}{2} m_e v^2 - \frac{1}{2} m_e v_o^2 = -eE\,(0.040\,m)$$

$$v^2 - v_o^2 = -\frac{2eE}{m_e}(0.040\,m) \qquad v^2 = v_o^2 - \frac{2(1.602\times10^{-19})12\times10^{3}(0.040)}{9.11\times10^{-31}}$$

$$\boxed{v = 1.52\times10^{7}\ m/s}$$

- o 1.5×10^7 m/s
- o 1.2×10^7 m/
- o 2.4×10^7 m/
- o 2.1×10^7 m/
- o 1.8×10^7 m/

10 points

2. An irregular conductor carries a surface charge density of $-6.75\ \mu C/m^2$ at and in the vicinity of point *P* on the surface. An electron is released just above *P* outside the conductor. What is the magnitude of its acceleration the instant after it is released?

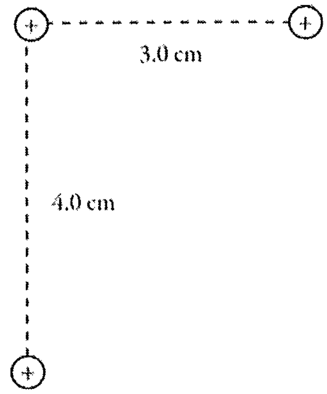
$$E = \frac{\sigma}{\varepsilon_o} \qquad F = qE \qquad ma = qE = q\frac{\sigma}{\varepsilon_o} \qquad a = \frac{q}{m}\frac{\sigma}{\varepsilon_o}$$

$$a = \frac{(-1.602\times10^{-19})(-6.75\times10^{-6})}{(9.11\times10^{-31})(8.85\times10^{-12})} = 1.34\times10^{17}\ m/s^2$$

Acceleration = 1.34×10^{17} m/s^2

10 points

3. Consider the group of three +5.1 nC point charges shown in the figure. What is the electric potential energy of this system of charges relative to infinity?



$$U = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{.03} + \frac{1}{.04} + \frac{1}{.05}\right)$$

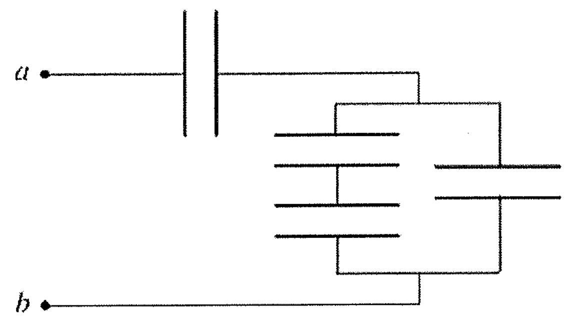
$$U = \frac{(5.1 \times 10^{-9} C)^2}{4\pi\, 8.85 \times 10^{-12}} (78.3)$$

$$U = 1.83 \times 10^{-5} \text{ joules}$$

PE = 1.83×10^{-5} joules

10 points

4. The capacitors in the network shown in the figure all have a capacitance of 5.0 μF. What is the equivalent capacitance, C_{ab}, of this capacitor network? **(circle the correct answer)**



$$\frac{1}{5\mu F} + \frac{1}{5\mu F} = \frac{2}{5\mu F} \qquad C_{eq}^{(1)} = \frac{5}{2}\mu F$$

$$C_{eq}^{(2)} = 7\tfrac{1}{2}\mu F = \frac{15}{2}\mu F$$

$$C_{eq}^{(3)} = \left(\frac{1}{5\mu F} + \frac{2}{15\mu F}\right)^{-1} = \left(\frac{5}{15\mu F}\right)^{-1} = 3\mu F$$

- o 3.0 μF
- o 5.0 μF
- o 20 μF
- o 10 μF
- o 1.0 μF

5 points

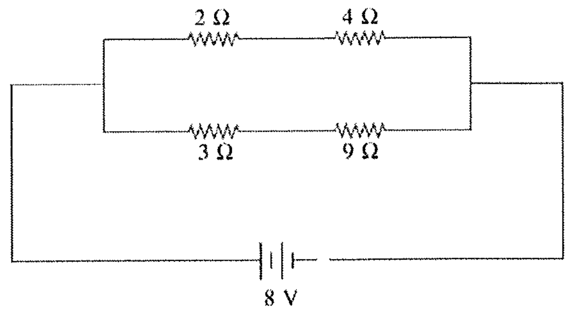
5. An electric furnace consumes 24 kW when it is connected to a 240-V line. What is the resistance of the furnace? **(circle the correct answer)**

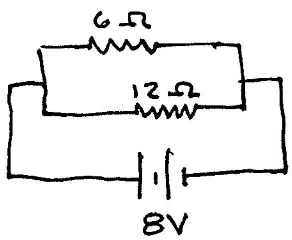
$$P = \frac{V^2}{R} \Rightarrow \quad R = \frac{V^2}{P} = \frac{(240 \text{ volts})^2}{24{,}000 \text{ W}} = 2.40\,\Omega$$

- o 2.4 Ω
- o 0.42 Ω
- o 2.0 Ω
- o 10 Ω
- o 100 Ω

10 points

6. Four resistors are connected across an 8-V DC battery as shown in the figure. The current through the 9-Ω resistor is closest to:
(circle the correct answer)





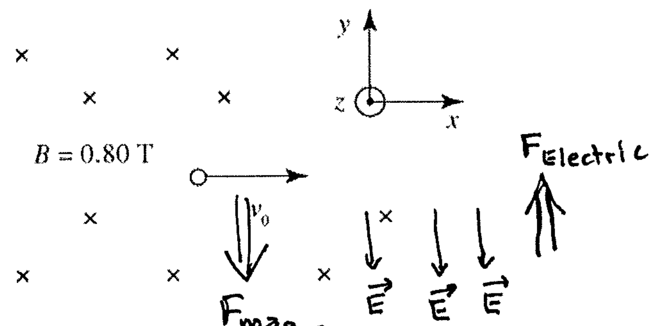
- o 0.5 A
- o 1.0 A
- o 0.9 A
- o 2.0 A
- o 0.7 A

The current through the 9 Ω resistor is

$$I = \frac{V}{R} = \frac{8 \text{ volts}}{12\,\Omega} = \frac{2}{3}\text{ A}$$

10 points

7. A uniform magnetic field of magnitude 0.80 T in the negative z-direction is present in a region of space, as shown in the figure. A uniform electric field is also present. An electron that is project with an initial velocity $v_o = 9.5 \times 10^4$ m/s in the positive x-direction passes through the region without deflection. What is the electric field vector in the region?
(circle the correct answer)



$$q\vec{E} + q\vec{v}\times\vec{B} = \vec{0}$$

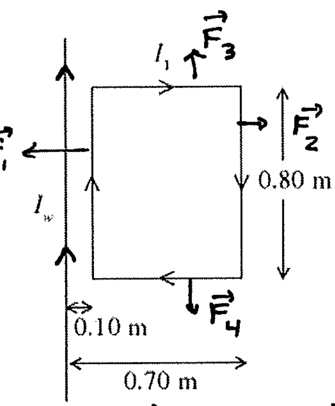
- o -76 kV/m $\hat{j}$
- o -120 kV/m $\hat{j}$
- o +120 kV/m $\hat{\imath}$
- o +120 kV/m $\hat{j}$
- o +76 kV/m $\hat{\imath}$

$$E = VB = 9.5 \times 10^4 \text{ m/s } (0.8\text{T})$$

$$E = 7.60 \times 10^4 \text{ Volts/meter}$$

10 points

8. As shown in the figure, a rectangular current loop is carrying current I_1=5A, in the direction shown, and is located near a long wire carrying a current I_W. The long wire is parallel to the sides of the rectangle. The rectangular loop has length 0.80m and its sides are 0.10m and 0.70m from the wire, as shown. We measure that the net force on the rectangular loop is 6.5 $\times 10^{-6}$ N and is directed towards the wire.



What is the magnitude of the current I_W? **(circle the correct answer)**

$$\vec{F} = I\vec{L}\times\vec{B}$$

$$\vec{F}_{TOTAL} = F_1 - F_2 = IL(B_1 - B_2)$$

- o 0.55 A
- o 0.95 A
- o 1.15 A
- o 3.25 A
- o 4.50 A

$$\vec{F}_{NET} = IL\frac{\mu_o I_w}{2\pi}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{\mu_o I I_w L}{2\pi}\left[8.57 \text{ m}^{-1}\right]$$

$$I_w = \frac{2\pi}{\mu_o}F_{NET}\frac{1}{IL[8.57\text{m}^{-1}]} = \frac{10^7(6.5\times10^{-6})}{2(5A)(0.8\text{ m})[8.57\text{m}^{-1}]}$$

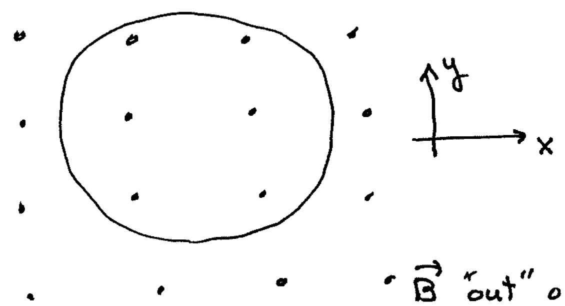
In which direction does I_W flow? **(circle the correct answer)**

- o From bottom to top
- o From top to bottom

10 points

9. A loop of radius r = 3.0 cm is placed parallel to the *xy*-plane in a uniform magnetic field $\vec{B} = 0.75\,T\,\hat{k}$. The resistance of the loop is 18Ω. Starting at *t=0,* the magnitude of the field decrease uniformly to zero in 0.15 seconds. What is the magnitude of the electric current produced in the loop during that time? **(circle the correct answer)**

$$\frac{dB}{dt} = \frac{-0.75}{0.15s} = -5\ T/sec$$



$\cdot\ \vec{B}$ "out" of the paper

$$\mathcal{E} = -\frac{dB}{dt} A \underbrace{\cos\Theta}_{=1}$$

- o 0.20 mA
- o 3.9 mA
- o 0.79 mA
- o 2.1 mA
- o 1.7 mA

$$\mathcal{E} = +5.0\ \frac{T}{sec}\left(\pi(.03m)^2\right)$$

$$\mathcal{E} = 1.41\times10^{-2}\ \text{volts}$$

$$I = \frac{\mathcal{E}}{R} = \frac{1.41\times10^{-2}\ \text{volts}}{18\,\Omega} = \underline{\underline{0.785\ mA}}$$

5 points

10. How much energy is stored in a room 3.0 m by 4.0 m by 2.4 m due to the earth's magnetic field with a strength of 5.0 $\times10^{-5}$ T? **(circle the correct answer)**

- o 100 mJ
- o 579 mJ
- o 29 mJ
- o 10 mJ
- o 570 mJ

$$u_B = \text{mag. energy density} = \frac{1}{2\mu_0} B^2$$

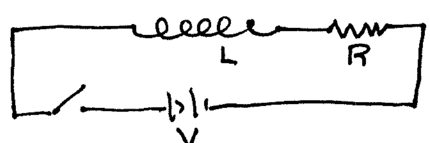
$$u_B = \frac{1}{8\pi\times10^{-7}}(5.0\times10^{-5}\,T)^2 = 9.95\times10^{-4}\ J/m^3$$

$$U_B = \text{mag. energy} = u_B\times(\text{Volume}) = 9.95\times10^{-4}\ J/m^3\,(28.8\,m^3)$$

$$\boxed{U_B = 28.7\times10^{-3}\ \text{joules}}$$

5 points

11. What resistance should be added in series with a 7.0-H inductor to complete an *LR* circuit with a time constant of 5.0 ms? **(circle the correct answer)**



- o 1.4 Ω
- o 35 Ω
- o 1.4 k Ω
- o 3.6 Ω
- o 15 k Ω

$$-L\frac{di}{dt} - iR = 0$$

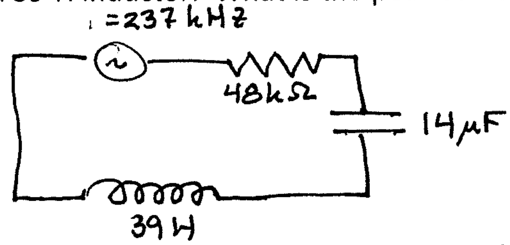
$$\frac{di}{dt} = -i\frac{R}{L} \Rightarrow \text{solution}\quad i(t) = I_0 e^{-\frac{R}{L}t}$$

$$\tau = \frac{L}{R} \qquad i(t) = I_0 e^{-t/\tau}$$

$$R = \frac{L}{\tau} = \frac{7.0\ H}{5\times10^{-3}\ s} = \boxed{1.40\ k\Omega}$$

10 points

12. A series LRC circuit has a sinusoidal voltage supplied to it at 237 kHz with a peak voltage of 866 V, a 48-kΩ resistor, a 14-µF capacitor, and a 39-H inductor. What is the peak current for this circuit? **(circle the correct answer)**



- o 15 µA
- o 11 µA
- o 21 µA
- o 18 µA
- o 9 µA

$I = \frac{V}{Z}$

$\omega = 2\pi f = 1.49 \times 10^{6}$ rad/s

$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = 58.1\ M\Omega$

$I = \frac{866 \text{ volts}}{581\ M\Omega} = \underline{\underline{14.9\ \mu A}}$

5 points (extra credit)

13. A series *LRC* ac circuit has a resistance of 4.0 kΩ, a capacitance of 33.0 µF, and an inductance of 23.0 H. If the frequency of the alternating current is 2.0/π kHz, what is the phase angle between the voltage and current? **(circle the correct answer)**

- o 3.1 rad
- o -0.52 rad
- o 0.83 rad
- o 1.5 rad
- o -1.6 rad

$\tan\phi = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}$

$\tan\phi = \frac{4\ kHz(23\ H) - \frac{1}{4\ kHz(33\ \mu F)}}{4\ k\Omega} = 23.0$

$\omega = 2\pi f$

$\omega = 2\pi\left(\frac{2}{\pi} kHz\right)$

$\omega = 4.0\ kHz$

$\phi = \tan^{-1}(23.0) = \underline{\underline{87.5^{\circ}}} \longrightarrow \underline{\underline{1.53 \text{ radians.}}}$