Physics I Equation Sheet

Dr. Darrel Smith

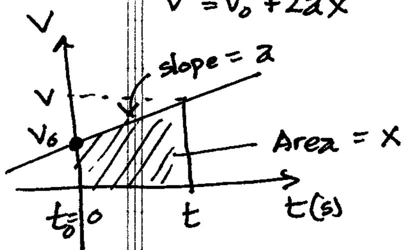
Kinematic (Constant acceleration only)

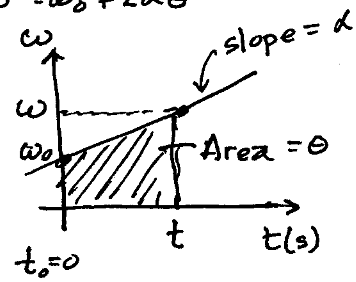
$v = v_0 + at$ $\quad \omega = \omega_0 + \alpha t$

$x = \frac{1}{2}(v_0 + v)t$ $\quad \theta = \frac{1}{2}(\omega_0 + \omega)t$

$x = v_0 t + \frac{1}{2}at^2$ $\quad \theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$v^2 = v_0^2 + 2ax$ $\quad \omega^2 = \omega_0^2 + 2\alpha\theta$

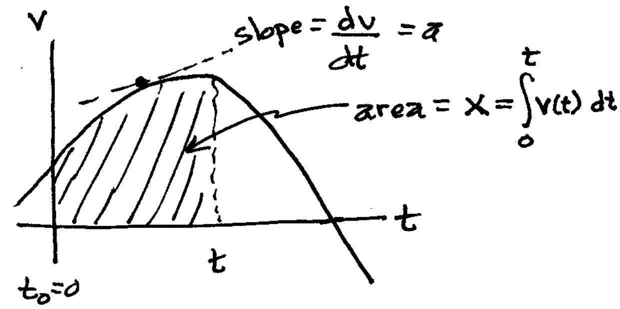




Non-constant acceleration

$v_{Av} = \frac{\Delta x}{\Delta t}$ $\quad \omega_{Av} = \frac{\Delta\theta}{\Delta t}$

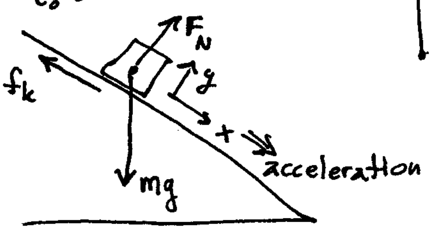
$a_{Av} = \frac{\Delta v}{\Delta t}$ $\quad \alpha_{Av} = \frac{\Delta\omega}{\Delta t}$



Dynamics

$$\sum \vec{F} = m\vec{a}$$

$$\begin{cases} \sum F_x = ma_x \\ \sum F_y = ma_y \\ \sum F_z = ma_z \end{cases}$$



$\sum F_y = 0 \Rightarrow F_N - mg\cos\theta = 0 \quad F_N = mg\cos\theta$

$\sum F_x = ma_x \Rightarrow mg\sin\theta - f_k = ma_x$

$mg\sin\theta - \mu_k mg\cos\theta = ma_x$

$a_x = g(\sin\theta - \mu_k\cos\theta)$

Static Friction $\cancel{0 \le \mu_s \le \mu_s}$ $0 \le f_s \le f_s^{max} = \mu_s F_N$

Kinetic Friction $f_k = \mu_k F_N$

Momentum: $\vec{p} = m\vec{v}$

If $\sum F_x = 0$ then $P_x^{before} = P_x^{after}$

If $\sum F_y = 0$ then $P_y^{before} = P_y^{after}$

If $\sum F_z = 0$ then $P_z^{before} = P_z^{after}$

$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt} m\vec{v} = m\frac{d\vec{v}}{dt} = m\vec{a}$

Work-Energy $W = \Delta K = K_f - K_i$

$W = \int_a^b \vec{F} \cdot d\vec{s}$ if force is not constant $= K_f - K_i$

$W = F \cdot \Delta s$ if force is constant $= K_f - K_i$

Work done by conservative forces: $W_{gr} = -\Delta U_{grav}$ $\quad W_{el} = -\Delta U_{el}$

$U_{grav} = mgy$ $\quad U_{el} = \frac{1}{2}kx^2$

where y must be vertically upward

Work done by non-conservative forces: $W_{nc} = \int \vec{F} \cdot d\vec{s}$

Work-Energy Theorem: $W_{TOTAL} = W_{CONS} + W_{NON-CONS} = \Delta K = K_f - K_i$

Kinetic Energy: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = K_{Trans.} + K_{rotation}$

Moment of Inertia: $I = \sum m_i r_i^2 \rightarrow I = \int dm\, r^2$

Discrete masses — Continuous mass

Parallel-Axis Theorem: $I_{new} = I_{cm} + Md^2$

Conservation of Mechanical Energy: $ME_{initial} = ME_{final}$

No friction, or other forces that dis

$\frac{1}{2}mv_i^2 + \frac{1}{2}I_i\omega_i^2 + mgy_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}I_f\omega_f^2 + mgy_f + \frac{1}{2}kx_f^2$

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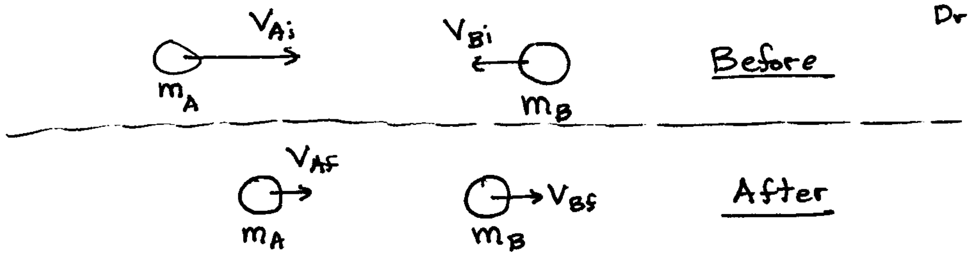
Elastic Collisions:

$K_f = K_i$

$\Delta K = 0$

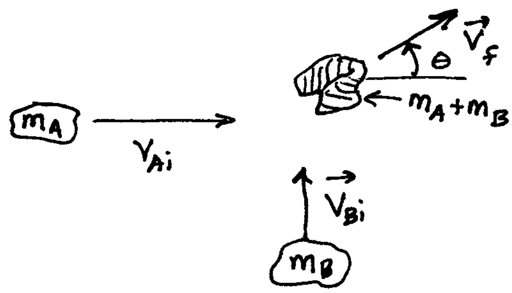


$$V_{Af} = \frac{m_A - m_B}{m_A + m_B} V_{Ai} + \frac{2m_B}{m_A + m_B} V_{Bi} \qquad V_{Bf} = \frac{2m_A}{m_A + m_B} V_{Ai} + \frac{m_B - m_A}{m_A + m_B} V_{Bi}$$

Inelastic Collisions:

$P_{before} = P_{after}$

$\vec{V}_f = V_{fx}\,\hat{\imath} + V_{fy}\,\hat{\jmath}$



① $m_A V_{Ai} = (m_A + m_B) V_{fx} = (m_A + m_B) V_f \cos\theta$

② $m_B V_{Bi} = (m_A + m_B) V_{fy} = (m_A + m_B) V_f \sin\theta$

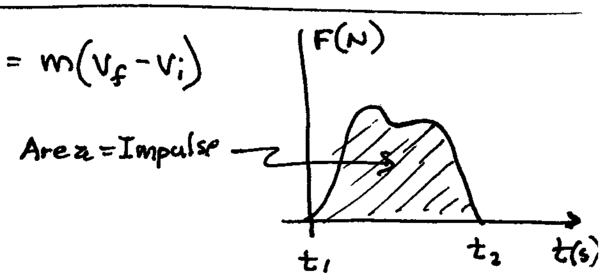
2 equations & 2 unknowns (V_f, θ)

② ÷ ① $\Rightarrow \tan\theta = \dfrac{m_B V_{Bi}}{m_A V_{Ai}} \qquad \theta = \tan^{-1}\left(\dfrac{m_B V_{Bi}}{m_A V_{Ai}}\right)$

$①^2 + ②^2 = m_A^2 V_{Ai}^2 + m_B^2 V_{Bi}^2 = (m_A + m_B)^2 V_f^2 \qquad V_f = \dfrac{\sqrt{m_A^2 V_{Ai}^2 + m_B^2 V_{Bi}^2}}{m_A + m_B}$

Impulse-Momentum Theorem

$$\text{Area} = \text{Impulse} = \int_{t_1}^{t_2} F \cdot dt = \Delta p = m\,\Delta v = m(v_f - v_i)$$



Impulse = change in momentum

$\int F\,dt = F\,\Delta t$ if F = constant or

$\int F\,dt = F_{Av}\,\Delta t$ if $F \neq$ constant.

Angular Momentum:

$\vec{L} \equiv \vec{r} \times \vec{p}$ point-particle

$\vec{L} = I\vec{\omega}$ Rigid Body

$\vec{\tau} \equiv \vec{r} \times \vec{F}$ def. of torque

$\tau_z = I_z \alpha_z$

$$\frac{d}{dt}\vec{L} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \underbrace{\vec{r} \times \frac{d\vec{p}}{dt}}_{\vec{r} \times \vec{F}} + \underbrace{\frac{d\vec{r}}{dt} \times \vec{p}}_{=0} = \vec{r} \times \vec{F} = \vec{\tau}$$

Similarities:

$$\Sigma \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt} \qquad \vec{p} = m\vec{v}$$

$$\Sigma \vec{\tau} = I\vec{\alpha} = I\frac{d\vec{\omega}}{dt} = \frac{d\vec{L}}{dt} \qquad \vec{L} = I\vec{\omega} = \vec{r} \times \vec{p}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = \hat{\imath}(yF_z - zF_y) + \hat{\jmath}(zF_x - xF_z) + \hat{k}(xF_y - yF_x)$$

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Center of Mass

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\Sigma m_i x_i}{\Sigma m_i}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\Sigma m_i y_i}{\Sigma m_i}$$

Power

P = power = rate at which work is done

Average $P = \dfrac{\Delta W}{\Delta t} = \dfrac{F\Delta s}{\Delta t} = F\, V_{Av}$ or $P_{Av} = \tau \dfrac{\Delta\theta}{\Delta t} = \tau\, \omega_{Av}$

Instantaneous Power $= P = \dfrac{dW}{dt} = F\dfrac{ds}{dt} = \vec{F}\cdot\vec{v}$

$$P = \frac{dW}{dt} = \frac{\tau\, d\theta}{dt} = \vec{\tau}\cdot\vec{\omega}$$

Conservation of Angular Momentum:

If $\Sigma \tau_z = 0$ then $(I_z\omega_z)^{Before} = (I_z\omega_z)^{After}$

If $\Sigma \vec{F} = \vec{0}$ then $\sum_i(\vec{r}_i \times m_i\vec{v}_i)_{before} = \sum_i(\vec{r}_i \times m_i\vec{v}_i)_{after}$ $\quad L^{Before} = L^{After}$

Interactions: If an interaction is occuring, then a force must be present:

4 Forces $\Rightarrow$
1. Gravity
2. Electromagnetic
3. Weak (Radioactivity)
4. Strong (Nuclear)

① $F\Delta s = \Delta K = K_f - K_i$

② $F\Delta t = \Delta p = m(v_f - v_i)$

③ $\tau\Delta t = \Delta L = I_f\omega_f - I_i\omega_i$

where $\tau = |\vec{r}\times\vec{F}|$

Projectile Motion near the earth's surface

$x = V_{ox}\, t = (V_o \cos\theta)t$

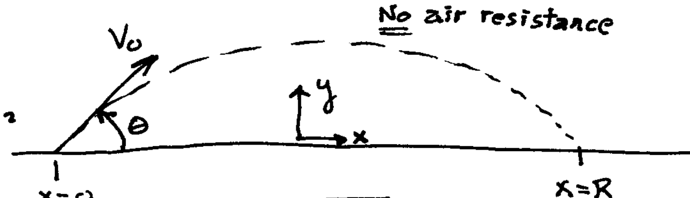
$V_x = V_{ox}$

$a_x = 0$

$y = V_{oy}\, t = (V_o \sin\theta)t - \frac{1}{2}gt^2$

$V_y = V_{oy} - gt$

$a_y = -g$



$$\boxed{R = \frac{V_o^2 \sin 2\theta}{g}}$$

Level Ground Only !!

Circular Motion $\quad s = R\theta \quad V_T = \dfrac{ds}{dt} = R\dfrac{d\theta}{dt} = \underline{R\omega} \quad a_T = \dfrac{d V_T}{dt} = R\dfrac{d\omega}{dt} = \underline{R\alpha}$

Radial Acceleration $a_r = \dfrac{V^2}{R} = R\omega^2 = \dfrac{4\pi^2}{T^2}R$

$V_T = \dfrac{\text{Circumference}}{\text{Period}} = \dfrac{2\pi R}{T} = R\omega \Rightarrow \boxed{\omega = \dfrac{2\pi}{T}}$

Rolling w/o Slipping

$V_{cm} = R\omega \qquad a_{cm} = R\alpha$

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Math Equations:

Quadratic Equation: $Ax^2 + Bx + C = 0$ $\quad x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

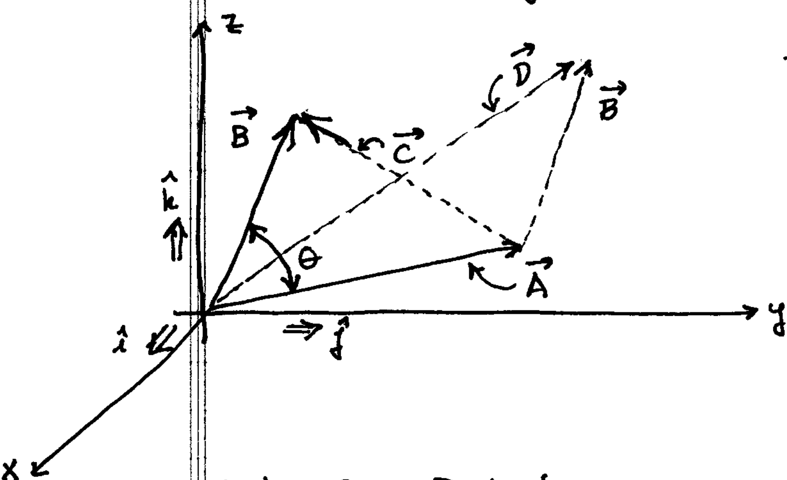
Vectors:

$\vec{A} = A_x \hat{\imath} + A_y \hat{\jmath} + A_z \hat{k}$

$\vec{B} = B_x \hat{\imath} + B_y \hat{\jmath} + B_z \hat{k}$

$\vec{C} = \vec{B} - \vec{A} = (B_x - A_x)\hat{\imath} + (B_y - A_y)\hat{\jmath} + (B_z - A_z)\hat{k}$

$D = \vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$



Scalar Dot Product

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta = AB\cos\theta$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{A\ B}$

$\vec{A}\cdot\vec{B} = (A_x B_x) + (A_y B_y) + (A_z B_z)$

$A = \sqrt{A_x^2 + A_y^2 + A_z^2} \qquad B = \sqrt{B_x^2 + B_y^2 + B_z^2}$

$A^2 = \vec{A}\cdot\vec{A} \qquad B^2 = \vec{B}\cdot\vec{B}$

Vector Cross Product

$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta \ \hat{n}$ ← unit vector following the Right-Hand Rule

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{\imath}\,(A_y B_z - A_z B_y) + \hat{\jmath}\,(A_z B_x - A_x B_z) + \hat{k}\,(A_x B_y - A_y B_x)$$

Unit Vectors

$\hat{\imath}\cdot\hat{\imath} = 1 \qquad \hat{\imath}\cdot\hat{\jmath} = 0 \qquad \hat{\imath}\cdot\hat{k} = 0 \quad \ldots$ etc

$\hat{\imath}\times\hat{\jmath} = \hat{k} \qquad \hat{\jmath}\times\hat{k} = \hat{\imath} \qquad \hat{k}\times\hat{\jmath} = -\hat{\imath} \quad \ldots$ etc.

$$\cancel{\hat{A} = \frac{\vec{A}\cdot\vec{A}}{A}} \qquad \hat{A} = \frac{\vec{A}}{A} = \frac{\vec{A}}{\sqrt{\vec{A}\cdot\vec{A}}} = \frac{A_x\,\hat{\imath}}{\sqrt{A_x^2 + A_y^2 + A_z^2}} + \frac{A_y\,\hat{\jmath}}{\sqrt{A_x^2 + A_y^2 + A_z^2}} + \frac{A_z\,\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

unit vector pointing in the direction of $\vec{A}$.