DATE

TOPIC

Lecture Chapter 9

Prob. 86 Neutron Star and Supernova Remnants

The Crab Nebula is a cloud of glowing gas ~ 10 light years across.
(1054 AD) P = power = 5×10^{31} W

$K_{ns} = \frac{1}{2} I_{ns} \omega^2$ $\omega = \frac{2\pi}{T}$ $I_{ns} = \frac{2}{5} MR^2$

Given: $T = 0.0331$ s $\frac{dT}{dt} = 4.22 \times 10^{-13} \frac{\text{sec.}}{\text{sec}}$

$$K_{ns} = \frac{1}{2} I_{ns} \left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2}{2} I_{ns} \frac{1}{T^2}$$

from chain-rule

$$\text{Power} = \frac{dK_{ns}}{dt} = 2\pi^2 I_{ns} \frac{d}{dt}\left(T^{-2}\right) = 2\pi^2 I_{ns} \left(-2T^{-3}\right)\left(\frac{dT}{dt}\right)$$

$$\text{Power} = -4\pi^2 \frac{I_{ns}}{T^3}\left(\frac{dT}{dt}\right) \qquad I_{ns} = \frac{\text{Power}\, T^3}{4\pi^2 \, dT/dt} = \frac{(5\times10^{31}\,\text{W})(0.0331\,\text{s})^3}{4\pi^2 (4.22\times10^{-13}\,\text{s/s})}$$

$$\boxed{I_{ns} = 1.088 \times 10^{38}\ \text{kg}\cdot\text{m}^2}$$

b.) $M_{ns} = 1.4$ solar masses $= 1.4\left(1.99 \times 10^{30}\ \text{kg}\right)$

$I_{ns} = \frac{2}{5} M_{ns} R^2$ $R^2 = \frac{5\, I_{ns}}{2\, M_{ns}}$ $R = \sqrt{\frac{5}{2} \frac{1.088\times10^{38}\ \text{kg}\cdot\text{m}^2}{1.4\left(1.99\times10^{30}\ \text{kg}\right)}}$

$\boxed{R = 9881 \text{ meters}}$ ~ 10 km

c.) $V_T = R\omega = R\frac{2\pi}{T} = 9881\text{ m}\left(\frac{2\pi}{0.0331\text{ s}}\right)$ $\boxed{V_T = 1.88\times10^6\ \text{m/s}}$

$\frac{V_T}{C} = \frac{1.88\times10^6\ \text{m/s}}{3.00\times10^8\ \text{m/s}} = 6.25\times10^{-3}$ $\boxed{0.625\%\text{ of } c}$

C → speed of light

d.) $\text{Density} = \frac{M_{ns}}{\text{Volume}} = \frac{1.4\left(1.99\times10^{30}\ \text{kg}\right)}{\frac{4}{3}\pi\left(9881\text{ m}\right)^3}$ $\boxed{\text{Density} = 6.89\times10^{17}\ \text{kg/m}^3}$