

Chapter 11

Equilibrium and Elasticity

1 Conditions for Equilibrium

1st condition for equilibrium

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0 \quad (1)$$

2nd condition for equilibrium

$$\sum \vec{\tau} = \vec{0} \quad (2)$$

2 Center of Gravity

$$x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$y_{\text{cm}} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$z_{\text{cm}} = \frac{m_1z_1 + m_2z_2 + m_3z_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

$$\vec{\mathbf{r}}_{\text{cm}} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2 + m_3\vec{\mathbf{r}}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = x_{\text{cm}}\hat{i} + y_{\text{cm}}\hat{j} + z_{\text{cm}}\hat{k} \quad (3)$$

$$\vec{\mathbf{r}}_{\text{cm}} = \frac{\sum_i m_i\vec{\mathbf{r}}_i}{m_1 + m_2 + m_3 + \cdots = M}$$

The sum of the torques under the influence of gravity can be written as:

$$\vec{\tau} = \sum_i \vec{r}_i \times \vec{w}_i = \sum_i \vec{r}_i \times m_i \vec{g} = \left(\sum_i m_i \vec{r}_i \right) \times \vec{g} = \vec{r}_{\text{cm}} \times M \vec{g} = \vec{r}_{\text{cm}} \times \vec{w}$$

What does this mean?

When calculating the torque (τ) for an extended body, and the torque is due to gravity, you can calculate the torque as if the all the mass were concentrated at the object's *center of mass*.

3 Solving Rigid-Body Equilibrium Problems

Prob. 6 Two people are carrying a uniform wooden board that is 3.00 m long and weights 160 N. If one person applies an upward force equal to 60 N at one end, at what point does the other person lift?

Prob. 10 A uniform ladder 5.0 m long rests against a frictionless, vertical wall with its lower end 3.0 m from the wall. The ladder weighs 160 N. The coefficient of static friction between the foot of the ladder and the ground is 0.40. A man weighing 740 N climbs slowly up the ladder. a) What is the maximum frictional force that the ground can exert on the ladder at its lower end? b) What is the actual frictional force when the man has climbed 1.0 m along the ladder? c) How far along the ladder can the man climb before the ladder starts to slip?

4 Stress, Strain, and Elastic Moduli

$$\frac{\text{Stress}}{\text{Strain}} = \text{Elastic Modulus} \quad (\text{Hooke's Law})$$

4.1 Tensile and Compressive Stress and Strain

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F_{\perp} \ell_o}{A \Delta \ell}$$

Introduce the units of pressure (F/A), the **pascal**.

$$1 \text{ Pa} = 1 \text{ N/m}^2 \qquad 1 \text{ psi} = 6895 \text{ Pa}$$

Examples of the Young's Modulus. Table 11.1

Ex. 25 A circular steel wire 2.00 m long must stretch no more than 0.25 cm when a tensile force of 700 N is applied to each end of the wire. What minimum diameter is required for the wire?

4.2 Bulk Stress and Strain

Definition of pressure: $p = F_{\perp}/A$

$$\text{Bulk Modulus} = B = \frac{\text{Bulk stress}}{\text{Bulk strain}} = -\frac{\Delta p}{\Delta V/V_o} \quad (\text{Bulk modulus})$$

The bulk modulus B has the same units as pressure, namely Pa.

The reciprocal of the bulk modulus is called the **compressibility**, k .

$$k = \frac{1}{B} = -\frac{1}{V_o} \frac{\Delta V}{\Delta p}$$

Higher values of k , means that the material is easier to compress.

$$k_{\text{water}} = 45.8 \times 10^{-11} \text{ Pa}^{-1}$$

Ex. 34 In the Challenger Deep of the Marianas Trench, the depth of seawater is 10.9 km and the pressure is 1.16×10^8 Pa (about 1.15×10^3 atm).
 a) If a cubic meter of water is taken from the surface to this depth, what is the change in its volume? (Normal atmospheric pressure is about 1.0×10^5 Pa. Assume that k for seawater is the same as the freshwater value given in Table 11.2.)
 b) What is the density of seawater at this depth? (At the surface, seawater has density of 1.03×10^3 kg/m³. $k_{\text{water}} = 45.8 \times 10^{-11}$ Pa⁻¹)

4.3 Shear Stress and Strain

$$\text{Shear stress} = \frac{F_{\parallel}}{A}$$

$$\text{Shear strain} = \frac{x}{h}$$

$$S = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_{\parallel}/A}{x/h} = \frac{F_{\parallel}}{A} \frac{h}{x} \quad (\text{Shear modulus})$$

Homework – Chapter 11

Exercises: 1, 6, 10, 14, 23, 24, 32, 33

Problems: